

PhD Candidate: M. Fernanda Jongewaard de Boer

Advisor: Dr. Christopher Beetle

ABSTRACT

On the paraxial approximation in quantum optics

We examine how best to associate quantum states of a single particle to different modes of a narrowly collimated beam of classical radiation modeled in the paraxial approximation. Our analysis stresses the importance of the relationship between two inner products naturally arising in the problem. These are the inner product used to expand a general beam as a superposition of orthogonal modes in the paraxial approximation, on the one hand, and the canonical inner product on which the statistical interpretation of quantum (field) theory is founded, on the other. While several candidates for the sort of association between beam modes and single-particle quantum states have been proposed in the literature; here we argue that one of them is uniquely well suited to the task. Specifically, the mapping from beam modes to "henochromatic" fields on spacetime is unique within a large class of similar mappings in that it is unitary in a mathematically precise sense. We also show that the single-particle quantum states associated to the orthogonal modes of a classical beam in the henochromatic approach are not only orthogonal, but also complete in the quantum Hilbert space.