Research Interests: Wolfgang Tichy

My main expertise is in numerical simulations of the Einstein equations in the fully non-linear regime. In particular, I am working on simulations of binary black hole systems. I have experience in both the time evolution of such systems and also in the construction of realistic initial data. Modeling the inspiral and merger phase of black hole binaries is one of the major challenges in gravitational wave physics today. Advances in supercomputer technology, new formulations of the evolution equations, improved numerical methods, and more realistic initial data will make it possible to perform such simulations in the near future. The results of these simulations will most likely be used to extract and analyze signals of binary black hole inspirals and mergers from data acquired by interferometric detectors such as LIGO, GEO, VIRGO, TAMA and LISA. Apart from being a pressing problem from the data analysis viewpoint, the binary black hole problem is also of fundamental importance as it probes the fully non-linear regime of Einstein's equations. Both these aspects make it very exciting for me to work in this field. In addition my research interests also span a broad range of other topics in relativity and astrophysics including post-Newtonian theory, gravitational waves, radiation reaction, and semiclassical relativity. In my opinion it is very important to have a broad range of knowledge and interests. This "big picture" view facilitates cooperation with researchers in other areas of science.

Current and Past Research

Initial data for black hole inspirals

Mergers of two black holes with masses of $\sim 10 - 100 M_\odot$ will be observable by the ground based gravitational wave detectors. In order to predict their highly relativistic orbits and waveforms, fully nonlinear numerical simulations should be used. All such numerical simulations must begin by specifying initial data. I am very interested in constructing initial data that accurately represent astrophysical systems such as two black holes orbiting each other, because only then will a simulation be astrophysically relevant. Together with B. Brügmann, M. Campanelli and P. Diener, I have worked on a project to generate post-Newtonian based initial data for two inspiraling black holes. The idea is that even though post-Newtonian theory may not be able to evolve two black holes when they get close, it can still provide initial data for fully nonlinear numerical simulations, if we start at a separation where post-Newtonian theory is valid. However, before this data can be used in general relativistic simulations we have to modify it, since post-Newtonian theory in principle deals with point particles and not black holes. Also, the pure post-Newtonian data do not fulfill the constraint equations of General Relativity. To overcome these problems we first resum the post-Newtonian expansion in such a way that the data do contain black holes. In a second step we then project the data onto the solution manifold of General Relativity by solving the constraint equations [1] using the Cactus code. The resulting data do satisfy the constraint equations of General Relativity, and incorporate post-Newtonian features in the sense that they are close to the original post-Newtonian data.

Post-Newtonian calculations predict that black hole binaries are expected to move on quasi-circular orbits with a slowly shrinking radius, i.e. the binary system is in quasi-equilibrium. Together with B. Brügmann and P. Laguna, I have started to investigate how to find coordinate systems which corotate with the two orbiting black holes. Such coordinate systems have the advantage that the rapid circular motion of the two black holes is transformed away so that one has to simulate only the slower drift of the holes toward each other. It is hoped that then the numerical simulations will be more accurate and stable. In order to find such corotating coordinates for the initial data, we use several conditions which hold if the orbital timescale is much shorter than the inspiral timescale. In particular, we use necessary conditions for the existence of an approximate helical Killing vector, such as equality of Komar and ADM mass. As a first step we have applied these ideas [2] to standard puncture initial data, which are similar to, but simpler than, the post-Newtonian based initial data discussed above. We have also been able to use our conditions to construct sequences of quasi-equilibrium puncture initial data [3].

All numerical calculations described so far were carried out on finite spatial domains using finite differencing techniques. Recently M. Ansorg, B. Brügmann and I have implemented a new numerical method for the computation of puncture data. It involves a coordinate transformation, which compactifies all of space into a finite box. In these coordinates we are able to use a spectral method to numerically solve the initial data equations. The result is much more accurate and requires far less computational resources than the one obtained using conventional finite differencing techniques. We are working on using this accuracy to investigate puncture data in the small mass ratio limit. The aim is to compare puncture data in this limit with the known analytic solution of a point particle in circular orbit around a black hole. In this way it will be possible to assess the astrophysical quality of puncture data.

Black hole evolutions

Once we have computed the initial data for a binary black hole system, we are in principle set up to simulate their inspiral and merger. Unfortunately however, this area of research is plagued by numerical instabilities that have so far prevented long term simulations. For this reason I am very interested in formulations of the Einstein equations with better stability properties. First order symmetric hyperbolic formulations of the evolution equations are promising in this respect, as they are guaranteed to yield results which converge to the true solution at least for a finite time interval. However, even if an evolution system is symmetric hyperbolic there is no guarantee that its numerical implementation will be stable in the long run when we evolve a highly dynamic black hole spacetime. Thus any system of interest should be tested numerically before we can draw definite conclusions about its stability.

Recently I have been involved in implementing and testing two new evolution systems. The first one was devised by Alekseenko and Arnold. This system as a whole is not symmetric hyperbolic, however it possesses a symmetric hyperbolic subsystem. Together with N. Jansen and B. Brügmann I have implemented and tested the numerical stability of this system within the BAM code [4]. Unfortunately, our numerical experiments show that it is less stable than the well known Baumgarte-Shapiro-Shibata-Nakamura (BSSN) system when we evolve black holes in gauges which enhance stability, at least for the BSSN system.

B. Brügmann and I are currently investigating several strongly hyperbolic first order versions of the BSSN system. In particular, I have focused on a system devised by Frittelli and Reula, which is the first order extension of the BSSN system with the least number of variables. I have derived a parametrized transformation of variables which for a particular parameter choice brings the Frittelli-Reula system into a form that extends another first order version of BSSN due to Alcubierre, Brügmann, Miller and Suen, in effect unifying the two systems [5]. The transformation is designed such that it does not alter the hyperbolic properties of the original Frittelli-Reula system. I have also implemented this new system in the BAM code. Preliminary numerical results indicate that the system is not as stable as the original BSSN system. However our system has several free parameters and it is possible that stability will improve once we find the appropriate choice of parameters.

I am also actively involved in binary black hole evolutions using the more traditional BSSN system. I am working on extending the lifetime of our simulations by choosing a shift which brings the black hole binary into corotating coordinates. I have also worked on new techniques to excise the black hole interiors from the computational grid, so that we can better use the new types of initial data described above, which tend to be such that instabilities occur inside the black holes.

The simulations described so far were performed on a single computational grid with uniform resolution. However in order to resolve the black holes a certain minimum resolution is needed. Using this same resolution also away from the black holes is wasteful in terms of computer resources, so that it is for example impossible to put the outer grid boundary far from the black holes. For this reason it I am intrigued by the idea of mesh refinement, which allows several overlapping computational grids with different resolutions. Together with B. Brügmann, I am currently involved in an effort to introduce mesh refinement into the BAM code. Using several nested grids with different resolutions we are starting to perform evolutions of a single black hole, which seem to be just as stable as without mesh refinement.

Coordinate independent formulation of post-Newtonian theory

In many problems of interest in astrophysics it is impossible to solve the full Einstein equations of General Relativity analytically. For this reason the post-Newtonian expansion in powers of velocity over the speed of light has been developed. Usually, the equations of post-Newtonian theory are only given in a specific coordinate system (or gauge). In each of the many possible gauges the post-Newtonian equations take a different form, which has the disadvantage that it is often hard to compare calculations in different gauges. The situation is somewhat analogous to knowing electrodynamics only in a couple of different gauges without knowing the underlying gauge invariant Maxwell equations. É. Flanagan and I have started investigating the possibility of a gauge independent formulation of post-Newtonian General Relativity. At first post-Newtonian order, we have found such a theory [6]. It is formulated solely in terms of geometric objects, such as connections and tensors. We are able to show from our equations that the usual coordinate-dependent equations of post-Newtonian gravity can be recovered when one specializes to specific coordinates.

Post-Newtonian gravitational waveforms

The detection of gravitational waves from the inspiral of a compact binary depends crucially on the availability of accurate template waveforms, as the signal is likely to be buried in noise. If such a signal is to be detected using matched filtering, high accuracy is needed in the phase of the template. This phase is derived from the energy balance equation which states that the energy loss rate is equal to the gravitational wave luminosity of the binary. É. Flanagan, E. Poisson and I [7] have investigated if the accuracy of the templates' phase can be improved by solving the post-Newtonian energy balance equation exactly numerically, rather than (as is normally done) solving the energy balance equation analytically within the post-Newtonian perturbative expansion. We found evidence that there is no gain in accuracy. This result is disappointing, but constitutes useful information from the point of view of generating template banks for inspiral searches: there is no motivation in terms of increased event rate to solve numerically for the wave's phase.

Towards computing the radiation reaction force for test-particles in Kerr spacetime

The inspiral of compact stellar mass objects into supermassive black holes is an important source of gravitational waves which could be detected by the planned space-based LISA detector. Since the mass ratio is small, the compact object is well approximated by a test-particle. In order to find the inspiral orbit of a test-particle due to radiation reaction, É. Flanagan and I have tried to approximate the orbit as a sequence of geodesic orbits, with constants of the motion that slowly change on a radiation reaction timescale. The idea is to compute this adiabatic evolution of the constants of the motion from quantities such as energy and angular momentum and their fluxes at future null infinity. The first potential problem was that angular momentum is not necessarily well defined. However, we find [8] that angular momentum ambiguities arise only at higher order in the adiabatic expansions and can thus be neglected. A more serious problem is that we have shown [8] that the x- and y-components of the angular momentum flux vanish identically in the adiabatic approximation, so that energy and angular momentum alone do not contain enough information to compute the evolution of all constants of the motion for generic non-circular orbits around rotating (Kerr) black holes.

The expected stress-energy tensor of a massive scalar field

In semiclassical relativity, a classical metric is coupled to quantum fields. This is achieved by replacing the classical stress-energy tensor by its expected value in the Einstein equation. A difficulty in semiclassical relativity is the non-uniqueness of the expected stress-energy tensor. Wald has postulated a set of physically well motivated axioms or criteria, which any expected stress-energy tensor has to satisfy. For a massive scalar field É. Flanagan and I have shown that the ambiguity in the stress-energy tensor allowed by the Wald axioms is an infinite parameter ambiguity [9], which is different from the two parameter ambiguity known for massless scalar fields. In order to get an explicit example, we have also calculated the expected stress-energy tensor in the incoming vacuum state for a massive scalar field in a spacetime which is a linear perturbation of Minkowski spacetime. Somewhat surprisingly, the result of our specific calculation explicitly exhibits only a two parameter ambiguity, just as in the massless case, even though this is not guaranteed by the Wald axioms. It will be interesting to see if the Wald axioms can be extended to rule out more than two undetermined parameters in the massive case as well.

Future Research Plans

Astrophysically realistic initial data

Black hole inspirals and mergers will be observable by gravitational wave detectors. In the coming years numerical simulations will be used to predict their highly relativistic orbits and waveforms. However, the astrophysical relevance of numerical simulations depends crucially on the use of astrophysically realistic initial data. There are at least two conditions which the data have to fulfill in order to be realistic: (i) the data have to agree with post-Newtonian data far from the black holes, (ii) near the black holes they should agree with predictions from black hole perturbation theory, in particular the black holes should have the correct amount of tidal deformation. In addition it would be very useful to have the data in coordinates, which are corotating with the black holes. Our post-Newtonian data [1] already satisfy (i). I have several ideas on how to verify the quality of the initial data near each horizon by comparing with perturbed black hole results. For example, it will be important to determine the tidal deformation of the horizon. Preliminary calculations show that this tidal deformation should be very small for realistic data. It may also be useful to use the isolated horizon framework, introduced by Ashtekar and collaborators, to compute invariant quantities on the horizon. Together with quantities computed at infinity this may open a way to estimate the radiation content of the data. In this area there is a lot of opportunities for collaboration with mathematical relativists. I plan to use these different ideas to develop a set of tests, which can be used to assess the quality of initial data sets. I also want to use my knowledge about post-Newtonian theory to match perturbed black hole solutions to post-Newtonian solutions in order to obtain more realistic data. Combining different perturbation methods with numerical calculations may give valuable information about black hole binaries at intermediate separations and will provide better initial data for fully numerical simulations.

Another line of research I want to follow up on is to use the Cactus code to evolve our post-Newtonian data [1], and then to extract waveforms using the Lazarus approach pioneered by Baker, Brügmann, Campanelli and Lousto. I have just started to collaborate on this project with M. Campanelli at the Center for Gravitational Wave Astronomy in Brownsville. The idea is that even if our post-Newtonian data is not yet completely realistic in terms of astrophysical content, we want to get the infrastructure ready to extract waves from numerically evolved post-Newtonian data. At the same time I am also planning to add post-Newtonian waveforms to our initial data. If we then evolve numerically we might eventually be able to compute numerical waveforms, which continuously match post-Newtonian waveforms.

Fully non-linear evolutions

So far fully non-linear numerical evolutions of black holes in three dimensions have always been hampered by numerical instabilities. I think that it is a fascinating challenge to come up with improvements in this area. I plan to continue working on symmetric hyperbolic evolution systems for the Einstein equations in order to improve numerical stability. In addition I am interested in finding gauge conditions which are well adapted to the problem at hand, such as corotating coordinates for the binary black hole problem. There are however several other ideas I wish to explore. Einstein's equations naturally split into evolution equation and constraint equations. Usually one solves the constraint equations for the initial data. After that one only uses the evolution equations to evolve forward in time. Analytically this is perfectly reasonable as the evolution equations preserve the constraints. However numerical errors may cause the numerical solution to wander away from constraint fulfillment, resulting in a solution which does not satisfy all of Einstein's equations. Moreover such constraint violations often grow exponentially and cause the computer code to fail. I plan to investigate methods to suppress constraint violations. One possible avenue is to use my experience in solving elliptic equations to re-solve the constraints at least from time to time during the evolution. A second possibility is to introduce additional variables, such that some of the constraint equations become evolution equations for these new variables, with the effect that these constraints are then automatically satisfied during evolution. I think it is also possible to combine these two methods. Another possibility is to use a new system of equations due to Bona, Ledvinka, Palenzuela and Zacek, who extend Einstein's equations by introducing additional fields. Of course in order to get a real physical solution these fields have to vanish identically. The advantage however is that this new system has no more constraint equations, only evolution equations. The additional fields, which are closely related to the deviation from the constraints in the original theory, obey wave equations, so that it is straightforward to devise boundary conditions which prevent such fields from entering the computational domain, while at the same time allowing them to leave it. In this way one has more control over the constraint deviations. I plan to use a combination of these methods to get a more stable evolution system.

Finally, I am also interested in working on problems containing matter. For example I would like to work on simulations of neutron star binaries or binaries containing a black hole and a neutron star. At least in the latter case, my experience with mesh refinement may be helpful because of the difference in either size or field strength of the two objects. Also, as matter fields often develop shocks the evolution equations should be expressed in flux-conservative form. Here my knowledge about symmetric hyperbolic evolution may prove useful, as such systems can most easily be expressed in flux-conservative form. Another area I am interested in working on in the future is the general relativistic simulation of stellar core collapses, and the formation of neutron stars. In both these areas I see the potential for interdisciplinary cooperation with researchers in other areas of physics or astronomy.

In closing I would like to remark that I am also open to new ideas and willing to work in new areas of research.

Commitment to Teaching

Teaching experience

While still at the University of Karlsruhe I was a teaching assistant for graduate classes in classical and also quantum mechanics. My main task was to highlight important concepts and to explain homework problems on the blackboard to graduate students and to prepare solution for homework problems. As a physics graduate student at Cornell University I was a teaching assistant in many different classes, ranging from General Physics classes for non-physics majors to Modern Physics classes for students in the honors physics sequence. My main duties were to prepare material for the classes I had to teach and to answer any questions the students may have had. I also had to prepare and grade weekly or bi-weekly quizzes. In addition I had office hours, where the students could ask questions. And of course I had to grade exams and homeworks, and write up homework solutions. I have also taught so-called Coop sessions where I divided the students into groups to work on problems in teams. Similarly, I have taught lab sessions where the students had to work on experiments in groups. In either case I went from group to group to answer questions, to help, or to ask questions myself in order to see if the students understood important concepts. Also at Cornell I was a grader in a statistical mechanics class taught by Ashcroft. My task was to grade the student's exams and homeworks and to write up solutions for the homework problems to be handed out to the students. All this has given me a lot of first hand experience on how students learn and also on what may prevent them from learning. I am eager to apply this experience in the classroom.

Teaching philosophy

A good teacher should recognize individual differences among students, encourage students to evolve and develop their own ideas, motivate them and recognize their progress. In addition to conducting lectures and designing labs the teacher should be available for the students during office hours. They should feel free and welcome to ask questions and discuss matters concerning them.

In teaching it is essential that the students are motivated to be curious about the material. Otherwise it is very hard for them to learn. In order to motivate students, lectures should be interesting and structured such that the students can see that they are making progress. It is very important to explain the basic concepts first and to explain them well, before embarking on long lectures about calculational methods. I think it is better to explain one thing well instead of explaining a lot of different things superficially. This way the students better understand the material and will be more motivated. Also it is important to pause from time to time to give them a chance to ask questions, and if they do not have questions, it may be good to ask them a question. Another crucial point is to have the lecture well planned. I plan to make and hand out notes about everything I wish to cover.

Furthermore it is important to take the educational background of the students into account. Especially their mathematical background should be considered in teaching physics. For example if most of the students are not familiar with differential equations, it is probably not helpful to use such equations, unless there is enough time to thoroughly explain the necessary mathematical background.

Assigned homework should be carefully selected to illustrate the concepts or calculational methods, explained in lecture. I think homeworks should count toward the final grade so that the students are motivated to do them. Nevertheless, I think it is good to not swamp the students with work. The amount of homework should be chosen such that they have time to think and read about the material. Less is sometimes more.

I think that doing experiments in physics is helpful, especially if they are fun. The hands-on experience may motivate students. Yet, the experiments should be designed such that they only require knowledge of material already covered in lecture. I do not think that it is a good idea to introduce new concepts in a lab. The students should enjoy doing the experiments, in order to motivate them to understand the material. In no case should their time in the lab be wasted. Once I saw an experiment in an introductory optics class, where the students first had to spend an hour adjusting mirrors and lenses in order to see some hydrogen lines. Such experiments are demotivating and do not really teach the material they intend to teach.

Teaching interests

At the graduate level, I would like to teach classes about General Relativity, numerical methods, quantum mechanics or mathematical methods for physicists. At the intermediate level I would like to teach classes in modern physics, such as about particles and waves, quantum mechanics, special relativity or electrodynamics. Of course I am also happy to teach any physics class at the introductory level.

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