Binary black hole mergers: Large kicks for generic spin orientations

Wolfgang Tichy and Pedro Marronetti

Department of Physics, Florida Atlantic University, Boca Raton, Florida 33431, USA

(Received 14 March 2007; revised manuscript received 25 May 2007; published 10 September 2007)

We present results from several simulations of equal mass black holes with spin. The spin magnitudes are \( S/m^2 = 0.8 \) in all cases, but we vary the spin orientations arbitrarily, inside and outside the orbital plane. We find that in all but one case the final merged black hole acquires a kick of more than 1000 km/s, indicating that kicks of this magnitude are likely to be generic and should be expected for mergers with general spin orientations. The maximum kick velocity we find is 2500 km/s and occurs for initial spins which are antialigned in the initial orbital plane.

I. INTRODUCTION

Currently several laser-interferometric gravitational wave detectors such as TAMA [1], LIGO [2], and GEO [3] are already operating, while several others are in the planning or construction phase [4]. One of the most promising sources for these detectors are the inspirals and mergers of binary black holes. The gravitational waves radiated during the inspiral and merger of two black holes carry energy, linear momentum, and angular momentum fluxes. The radiated energy is positive and thus the system will always lose energy. Except in the case of certain highly symmetric situations, such as head-on collisions of equal mass nonspinning black holes, the system usually also loses angular momentum. Generic systems with either unequal masses or spins with no particular alignment will also radiate linear momentum, resulting in a nonzero residual velocity (also known as recoil or “kick”) of the final black hole. The magnitude of this recoil is important in a variety of astrophysical scenarios, such as the cosmological evolution of supermassive black holes [5–12] or the growth and retention of intermediate-mass black holes in dense stellar clusters [13–19]. For a binary in almost circular orbit, the direction of the instantaneous linear momentum flux rotates in the orbital plane with the angular velocity of the system. Thus, when the binary goes through one orbital period, the average linear momentum flux will be close to zero. The only net effect comes from the fact that the inspiral orbits are not perfect circles. Most of the kick is thus accumulated during the last orbit and subsequent plunge of the two holes, when the motion of the two holes is no longer quasicircular and the averaged linear momentum flux is much larger. Several analytical estimates of the kick velocity have been published in recent years [20–23]. All these estimates have been derived using approximations (such as post-Newtonian theory) which break down during the last orbit and the subsequent merger. Hence these calculations contain a high degree of uncertainty. In addition, there are calculations which employ the close limit approximation [24,25]. However, these calculations can only model the plunge well, and miss contributions from the last orbit. There is also an analytic calculation based on attaching quasinormal ringdown modes to post-Newtonian waveforms [26]. Furthermore, results from several numerical simulations have been published [27–35] recently. The resulting maximum kick velocities found so far are 175 km/s for nonspinning unequal mass black holes [29], 257 km/s for equal mass black holes with unequal spins aligned with the orbital angular momentum [31], and 2500 km/s for equal mass black holes with antialigned spins \( S/m^2 = \pm 0.8 \) in the orbital plane [33]. The latter value is so high that the final black hole would escape e.g. both from dwarf elliptical and spheroidal galaxies (with typical escape velocities of below 300 km/s) and from giant elliptical galaxies (2000 km/s) [5]. Extrapolations from simulations of black hole binaries with spins in the orbital plane predict velocities as high as 4000 km/s [34]. The question thus arises if such high kick velocities are generic for spinning black holes, or if the results of 2500 km/s or even 4000 km/s arise only if the initial spins are exactly antialigned and confined to the initial orbital plane. To answer this question we have performed several simulations of black hole binaries on initially approximately circular orbits with spins in arbitrary directions. We find that except for one case the kick velocities are well above 1000 km/s. Thus, kicks of this magnitude seem to be a generic feature of black hole mergers with spin around or above \( S/m^2 = 0.8 \).

II. NUMERICAL TECHNIQUES

Our simulations are performed using the “moving punctures” method [36,37] with the BAM code [38,39] which allows us to use moving nested refinement boxes. We use 10 levels of 2:1 refinements. The outer boundaries are located 240M away from the initial center of mass \( M \) being the sum of the initial black hole masses), and our resolution ranges between \( 9M \) on the outermost box to \( M/56.9 \) near the black holes. Since our simulations have no particular symmetries, we cannot use the usual memory saving techniques and simulate only one quadrant of the numerical domain. Thus, our simulations take 4 times more memory and time than e.g. the ones reported in
This makes standard convergence tests very costly. Thus, the goal of this work is merely to give kick velocity estimates which can be used in astrophysics, instead of extremely accurate results. For the same reason, we present a convergence test for only one particular case in order to establish that our numerical setup gives results that are in the convergent regime. In Fig. 1 we show the energy radiated during the merger of a binary where both initial spins have magnitude $S/m^2 = 0.8$ and are antialigned to the orbital angular momentum. This radius should ideally be far from the sources. From the upper panel we see that the results for the three resolutions are consistent and very close to each other. The lower panel shows the difference between the low and medium, and also between the medium and high resolutions. The latter difference has been scaled such that the two curves would coincide for fourth order convergence. Assuming approximately fourth order convergence it would be possible to obtain error bars due to the finite differencing scheme used here. These error bars would be on the order of a few percent. Notice, however, that there are other sources of errors, the biggest of which is the extraction of the radiated energy and momentum at a finite radius. This radius should ideally be far from the sources. However, the outer regions of our grid are not very well resolved, since the highly refined boxes do not extend far from each black hole. We find that the best compromise for our grid structure is to extract at $r_{ex} \sim 30M$. Figure 2 shows the radiated momentum in the $z$ direction extracted at different radii. In this example both initial spins are perpendicular to each other and lie in the initial orbital plane. The first three curves (thick lines) at $r_{ex} = 20M$, $30M$, and $40M$ come from a run with our standard grid setup. In this case the grid resolutions at these radii are $h_{ex} = 1.125M$ at the two inner radii but only $h_{ex} = 2.25M$ at $r_{ex} = 40M$. This explains why the difference between the $30M$ and $40M$ results is not smaller than between the $30M$ and $20M$ results. We have also performed a second run (thin lines) where we have enlarged some of our refinement boxes. In this run $r_{ex} = 40M$ is now located on a level with $h_{ex} = 1.125M$ as well. If we compare the results $r_{ex} = 20M$, $30M$, and $40M$ for $h_{ex} = 1.125M$, we see that the difference between the $30M$ and $40M$ results is now smaller than the difference between the $30M$ and $20M$ results, as expected. In the run with the enlarged refinement boxes, the result for $r_{ex} = 30M$ remains unchanged (at the level of accuracy of this plot) because the resolution at $r_{ex} = 30M$ is unchanged, while there is a small difference for the $r_{ex} = 20M$ result (thin dashed line), because this radius is now located on an even better resolved level. The differences between these results at different radii and resolutions provide an estimate of the error due to extraction at a finite extraction radius. As a conservative estimate we use the average relative difference between the $30M$ and $40M$ results (at $t = 300M$) over all our simulations with the standard grid setup. The magnitude of this error estimate is approximately 20%. We will use this estimate.
also as an approximate error bar for the kick velocities presented below, since this error is expected to be dominant.

III. RESULTS

In order to gauge possible kick sizes we have performed several simulations with different spin orientations. In each case the spin magnitude of each black hole is \( S/m^2 = 0.8 \) and the initial orbital angular momentum is along the \( z \) axis. The magnitude of the orbital angular momentum is chosen such that the orbits are circular according to post-Newtonian approximations [40–42]. From previous works [43–49] it follows that this approach of setting initial data puncture parameters using post-Newtonian approximations is indeed possible. The bare mass parameter \( m_b \) of each puncture is chosen such that the Arnowitt-Deser-Misner (ADM) mass \( m \) measured at each puncture is approximately \( 0.5M \). Because of these approximations, it is clear that the orbits will have small ellipticities. However, since we are just interested in obtaining a sample of possible kick velocities for different initial conditions, such imperfections are not critical.

We label each simulation using eight numbers: \( [q, D, S_1/m^2, 90° - \theta_1, \phi_1, S_2/m^2, 90° - \theta_2, \phi_2] \). Defining the larger black hole as \#2, we set \( q = m_2/m_1 \geq 1 \). \( D \) and \( S_i/m^2 \) represent the holes’ coordinate separation and the spin magnitude of hole \( A (A = 1, 2) \). The angles \( \theta_A \) and \( \phi_A \) (given in degrees) determine the direction of each spin by their polar angles with respect to a coordinate system \( (\hat{x}, \hat{y}, \hat{z}) \), where \( \hat{z} \) is the unit vector in the direction of the binary’s initial orbital angular momentum, \( \hat{x} \) is the unit vector in the direction of the initial linear momentum of the larger hole, and \( \hat{y} = \hat{z} \wedge \hat{x} \) [50]. Since our runs all have equal masses and spin magnitudes, they are all of the form \( [1, 6M, 0.8, 90° - \theta_1, \phi_1, 0.8, 90° - \theta_2, \phi_2] \). The runs in Table I are thus labeled by the angles alone.

From Table I we see that almost all initial spin orientations lead to large kick velocities. The only exception is when both initial black hole spins point in exactly the same direction (see run \(-31, 0°, -31, 0\)). In this special case no net linear momentum is radiated (in agreement with post-Newtonian predictions [41]). We find that the largest kick occurs for antialigned spins that are perpendicular to the initial angular momentum \( J_i^\text{ADM} \). However, we also find substantial kicks for other initial spin orientations, and in each case the kick is well above 500 km/s. This means that kicks of this magnitude should be expected for generic black hole mergers with spins of magnitude of \( S/m^2 = 0.8 \) or above.

This result can be explained in the following way: it is known that the black hole spins significantly influence the dynamics of the two black holes. For example, the merger can be delayed if both spins are aligned with the orbital angular momentum [51]. Or the orbits can be nonplanar as in the case of a test mass in orbit around a spinning Kerr black hole [52]. Since the linear momentum radiated depends on the orbit shape, the kick velocity also depends

<table>
<thead>
<tr>
<th>Run</th>
<th>0, 90/0, -90</th>
<th>-1, 135/-1, 135</th>
<th>45, 0/-45, 180</th>
<th>44, 180/-46, 180</th>
<th>-31, 0/-31, 0</th>
<th>-1, 180/89, 180</th>
<th>-1, 50/89, 151</th>
<th>-1, 120/89, 119</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_i^\text{ADM}/M )</td>
<td>0.985</td>
<td>0.985</td>
<td>0.985</td>
<td>0.985</td>
<td>0.985</td>
<td>0.984</td>
<td>0.984</td>
<td>0.984</td>
</tr>
<tr>
<td>( J_i^\text{ADMD}/M^2 )</td>
<td>0.842</td>
<td>0.882</td>
<td>0.839</td>
<td>0.881</td>
<td>0.756</td>
<td>1.016</td>
<td>1.016</td>
<td>1.016</td>
</tr>
<tr>
<td>( m_b/M )</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>( m_b/M )</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>( D/M )</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
<td>6.000</td>
</tr>
<tr>
<td>( P/M )</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
<td>0.140</td>
<td>0.147</td>
<td>0.133</td>
<td>0.133</td>
<td>0.133</td>
</tr>
<tr>
<td>( L/M^2 )</td>
<td>0.842</td>
<td>0.840</td>
<td>0.839</td>
<td>0.839</td>
<td>0.881</td>
<td>0.798</td>
<td>0.798</td>
<td>0.798</td>
</tr>
<tr>
<td>( S_{1z}/m^2 )</td>
<td>0</td>
<td>-0.566</td>
<td>0.566</td>
<td>-0.576</td>
<td>0.684</td>
<td>-0.800</td>
<td>-0.693</td>
<td>-0.400</td>
</tr>
<tr>
<td>( S_{1z}/m^2 )</td>
<td>0</td>
<td>0.566</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.400</td>
<td>0.693</td>
</tr>
<tr>
<td>( S_{2z}/m^2 )</td>
<td>0</td>
<td>-0.566</td>
<td>-0.566</td>
<td>-0.566</td>
<td>0.684</td>
<td>-0.800</td>
<td>-0.693</td>
<td>-0.400</td>
</tr>
<tr>
<td>( S_{2z}/m^2 )</td>
<td>-0.800</td>
<td>-0.566</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.005</td>
<td>0.009</td>
</tr>
<tr>
<td>( M_f/M )</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>( J_f/M_f )</td>
<td>0.67</td>
<td>0.72</td>
<td>0.68</td>
<td>0.73</td>
<td>0.64</td>
<td>0.81</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>( v ) (km/s)</td>
<td>2500</td>
<td>1350</td>
<td>580</td>
<td>1350</td>
<td>0</td>
<td>1850</td>
<td>2100</td>
<td>1950</td>
</tr>
<tr>
<td>( \pm 500 )</td>
<td>( \pm 250 )</td>
<td>( \pm 150 )</td>
<td>( \pm 250 )</td>
<td>( \pm 50 )</td>
<td>( \pm 350 )</td>
<td>( \pm 450 )</td>
<td>( \pm 400 )</td>
<td></td>
</tr>
</tbody>
</table>
strongly on spin. In fact, if the spins are not parallel to the
orbital angular momentum, the spins and the orbital angu-
lar momentum both precess so that the resulting orbits do
not stay in one plane and become much more complex than
the familiar Newtonian circular orbits. Since the instanta-
neous linear momentum flux changes direction in a less
regular way than for circular orbits, one expects that the
averaged linear momentum flux is significantly larger than
for quasicircular orbits. As an example of such a more
complicated orbit, see Fig. 3 which shows the orbits of both
black hole centers for the case where one of the spins starts
out in the initial orbital plane and the other one perpen-
dicular to this plane. We can clearly see that the orbits do
not stay in one plane, and that the kick moves the final
merged black hole downward out of the initial orbital
plane.

From Figs. 1 and 2 we can see that the bulk of the
radiated energy and momentum is emitted within a time
of about $40M$. This is also the time during which the
maximum amplitude of the gravitational waves generated
by the merger passes through the detector. That is, almost
the entire kick is accumulated during the final merger
phase. Table I gives estimates for the mass and spin of
the final black hole.

Note that for the runs listed in Table I the angle between
the spins is either nearly $90^\circ$ or antialigned. Judging from
the aligned $-31, 0/ -31, 0$ case, we expect closer align-
ments to have smaller kicks.

In the $-1, 180/89, 180, -1, 150/89, 151$, and $-1, 120/89, 119$ runs, the initial spin of black hole #2 is nearly
perpendicular to the orbital plane, and it is almost the same
in each case. The initial spin of hole #1 is approximately in
the orbital plane, but the orientation differs by $30^\circ$ between
the $-1, 180/89, 180$ and $-1, 150/89, 151$ runs and by $60^\circ$
between the $-1, 180/89, 180$ and $-1, 120/89, 119$ runs.
We see that this change in orientation affects the kick
velocity. This effect can be explained if we use the post-
Newtonian formula (3.31b) in [41]. From this formula we
see that the instantaneous rate of radiated momentum
depends on the angle between the spin and the relative
velocity vectors, as well as on the angle between the spin
and separation vectors. Hence if we start out with different
spin orientations, we expect different spin orientations also
in the final merger phase where most of the final kick is
accumulated, which in turn leads to different amounts of
radiated momentum. A similar dependence on spin ori-
entation is described in [34] for a system with antialigned
spins in the initial orbital plane.

IV. DISCUSSION

We have performed simulations of equal mass black
holes with initial spins of magnitude of $S/m^2 = 0.8$. We
have tested several initial spin orientations and found that
all but one lead to kick velocities well above $1000$ km/s.
Thus kicks between $1000$ km/s and $2000$ km/s seem to be
a generic feature for spins of magnitude 0.8 with general
orientations. Higher spins will lead to even larger kicks.
This means that, for these cases, the final black hole will
likely escape from dwarf elliptical and spheroidal galaxies
(with escape velocities below $300$ km/s), and possibly
even from giant elliptical galaxies (with escape velocities
around $2000$ km/s) [5]. This result also has important
implications for a variety of astrophysical scenarios, such
as the cosmological evolution of supermassive black holes
or the growth and retention of intermediate-mass black
holes in dense stellar clusters. Note, however, that a more
extensive parameter search should be carried out before
drawing any definitive conclusions. Furthermore, notice
that all our simulations are for the equal mass case. It is
clear that the kick velocity will depend on the mass ratio
(for two proposed models see [32,35]), and we expect
smaller kicks for the unequal mass case. The probability
for the large kicks obtained here might be quite low, if kick
velocities drop sufficiently fast for mass ratios different
from 1. In addition, it has been suggested [53] that in real
galaxy mergers torques from accreting gas align the spins
with the orbital angular momentum, so that the kick ve-
locity is below $200$ km/s.

Almost the entire kick is accumulated during the final
merger phase, meaning that small changes in the initial
conditions are likely to lead to quite different kick direc-
tions and magnitudes. The reason is that a small change in
the initial velocities or separation (and also accumulated
numerical errors in the orbital phase) will significantly
alter the merger time [39], and thus alter the spin orienta-
tions just before merger. We therefore expect different
trajectories just before the merger which then will lead to
different final kicks. Note, however, that this sensitivity to
initial conditions does not change our main conclusion that
generic spin orientations can lead to very large kicks in the
equal mass case.
BINARY BLACK HOLE MERGERS: LARGE KICKS FOR...

ACKNOWLEDGMENTS

It is a pleasure to thank B. Brügmann, J. González, M. Hannam, S. Husa, and U. Sperhake for helpful comments and code development. This work was supported by NSF Grant No. PHY-0555644. We also acknowledge partial support by the National Computational Science Alliance under Grant No. PHY060021P and No. PHY060040T. Simulations were also performed at the Charles E. Schmidt College of Science computer cluster Boca 5.

[50] In this paper, we constructed our initial data sets such that the numerical grid coordinate system \((x, y, z)\) coincides with \((\hat{x}, \hat{y}, \hat{z})\).