Physics 5115 | Florida Atlantic University
Mathematical Physics | Fall, 2007

Problem Set V

Due: Monday, 29 October, 2007

1. The displacement of an infinitely long beam on an elastic foundation is described by

$$\left(\frac{\mathrm{d}^4}{\mathrm{d}x^4} + 1\right) y(x) = P(x),$$

where we assume that $y(\pm\infty) = 0$. Use the Fourier transform method to compute the Green function for this system. Then provide an integral which can be used to compute the displacement for an arbitrary load $P(x)$.

2. Heat diffuses in a medium that also radiates heat at a rate proportional to temperature. Thus, a one-dimensional spatial distribution of temperature $T(x,t)$ satisfies

$$\frac{\partial T}{\partial t} = D\,\frac{\partial^2 T}{\partial x^2} - \alpha\, T,$$

where D and α are positive constants. A finite amount of heat is released at a point in an infinite bar such that

$$T(x, t = 0) = Q\,\delta_0(x).$$

   a. Evaluate *and describe* the temperature distribution at later times.
   b. Compute the total amount of heat lost to radiation between time zero and time t, and interpret the result.

3. Demonstrate the equivalence between the representations

$$f(t) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{\sqrt{2\pi}}\, \hat{f}(\omega)\, \mathrm{e}^{-\mathrm{i}\omega t} = \frac{2}{\pi} \int_0^{\infty} \mathrm{d}\omega\, \left(\tilde{f}_{\mathrm{c}}^{+}(\omega)\, \cos\omega t + \tilde{f}_{\mathrm{s}}^{-}(\omega)\, \sin\omega t\right),$$

where

$$f^{\pm}(t) := \frac{f(t) \pm f(-t)}{2}$$

are the even and odd parts of $f(t)$, whose Fourier cosine and sine transforms are

$$\tilde{f}_{\mathrm{c}}^{+}(\omega) := \int_0^{\infty} \mathrm{d}t\, f^{+}(t) \cos\omega t \qquad \text{and} \qquad \tilde{f}_{\mathrm{s}}^{-}(\omega) := \int_0^{\infty} \mathrm{d}t\, f^{-}(t) \sin\omega t,$$

respectively. Relate these to the real and imaginary parts of $\hat{f}(\omega)$.