Physics 5115 | Florida Atlantic University
Mathematical Physics | Fall, 2007

Problem Set IV

Due: Monday, 15 October, 2007

1. Use induction to demonstrate that

$$x^m\,\delta^{(n)}(x) = \begin{cases} (-1)^m\,\frac{n!}{(n-m)!}\,\delta^{(n-m)} & n \geq m \geq 0 \\ 0 & m > n \geq 0 \end{cases}$$

in the operational sense of a generalized function integrated against a suitable function $f(x)$. What conditions must be imposed upon $f(x)$?

2. Suppose that a string is clamped at $x = 0$ and $x = L$ is under tension T and is subjected to a static load $F(x)$. The displacement $y(x)$ satisfies

$$T\,y''(x) = F(x)$$

for sufficiently small F/T. Note that F is force per unit length. Evaluate the Green function $G_y(x)$ satisfying

$$T\,\frac{\partial^2}{\partial x^2}\,G_y(x) = \delta_y(x)$$

and express the general solution for an arbitrary load distribution in terms of that Green function.

3. Use the computer (`Maple`, `MatLab`, `Mathematica`, or even `Excel` or `C`) to produce the plots called for in this problem.

   a. Use the Green function for a damped oscillator to solve

$$\ddot{x}(t) + 2\gamma\,\dot{x}(t) + \omega_0^2\,x(t) = f_0\,\Theta_0(t)$$

   for a step function. Find explicit solutions for the underdamped, overdamped, and critically damped cases. Compare these three solutions graphically and explain their behavior.

   b. Combine two of these solutions to determine the response to a square pulse

$$\ddot{x}(t) + 2\gamma\,\dot{x}(t) + \omega_0^2\,x(t) = f_0\,\big(\Theta_0(t) - \Theta_\tau(t)\big).$$

   Plot the solutions for $\gamma/\omega_0 = 0.5$, 1.0 and 2.0 with $\tau = 10/\omega_0$ as functions of $\omega_o\,t$ and explain their general characteristics. If this were an RLC circuit, under what conditions would the output follow the input most closely?