Review Problems VI

Recommended Reading: Chow, pp. 233–238.

1. (Chow 6.2) Given the complex numbers

\[ z_1 := \frac{3 + 4i}{3 - 4i} \quad \text{and} \quad z_2 := \left(\frac{1 + 2i}{1 - 3i}\right)^2, \]

find their polar forms, complex conjugates, moduli, product, and quotients.

2. (Chow 6.3) The absolute value or modulus of a complex number \( z := x + iy \) is defined as

\[ |z| = \sqrt{zz^*} = \sqrt{x^2 + y^2}. \]

If \( z_1 \) and \( z_2 \) are complex numbers, show that:

a. \(|z_1 z_2| = |z_1| |z_2|\),

b. \(|z_1/z_2| = |z_1|/|z_2|\) for \( z_2 \neq 0\),

c. \(|z_1 + z_2| \leq |z_1| + |z_2|\), and

d. \(|z_1 - z_2| \geq ||z_1| - |z_2||\).

3. (Chow 6.4) Find all possible complex values of

\[ z_{(a)} := \sqrt[4]{-32} \quad \text{and} \quad z_{(b)} := \sqrt[4]{1 + i}, \]

and plot them in the complex plane.

4. (Chow 6.5) Use De Moivre’s theorem to show that

a. \( \cos 5\theta = 16 \cos^5 \theta - 20 \cos^3 \theta + 5 \cos \theta \), and

b. \( \sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta \).