Problem Set V
Due: Tuesday, 5 October 2010

1. (Problems 2.9.12, 18 and 23, pp. 63–64)
Express each of the following complex numbers in the Cartesian form $z = x + iy$.

\[ z^{(a)} := 4 e^{-8\pi i/3}, \quad z^{(b)} := \left(\frac{1 + i}{1 - i}\right)^4 \quad \text{and} \quad z^{(c)} := \frac{(1 + i)^{48}}{(\sqrt{3} - i)^{25}}. \]

2. (Problems 2.10.10, 19 and 23, pp. 66–67)
Find all complex values of each of the following roots.

\[ z^{(a)} := \sqrt[3]{32}, \quad z^{(b)} := \sqrt[3]{i} \quad \text{and} \quad z^{(c)} := 4\sqrt[4]{8i}. \]

3. (Problems 2.11.4, 7 and 10, p. 69)
Find all complex values of each of the following functions.

\[ z^{(a)} := e^{3 \ln 2 - i\pi}, \quad z^{(b)} := \tan(i \ln 2) \quad \text{and} \quad z^{(c)} := \sin(i \ln i). \]

4. (Problems 2.12.2, 3, 8, 12 and 16, p. 71)
Verify each of the following identities using the definitions of the standard trigonometric and hyperbolic functions of a complex variable $z = x + iy$.

a. $\cos z = \cos x \cosh y - i \sin x \sinh y$

b. $\sinh z = \sinh x \cos y + i \cosh x \sin y$

c. $\cosh 2z = \cosh^2 z + \sinh^2 z$

d. $\cos^4 z + \sin^4 z = 1 - \frac{1}{4} \sin^2 2z$

e. $\tan iz = i \tanh z$

5. (Problems 2.11.27, 29 and 36, p. 71)
Evaluate each of the following functions in the Cartesian form $z = x + iy$

\[ z^{(a)} := \sin(4 + 3i), \quad z^{(b)} := \cosh 2\pi i \quad \text{and} \quad z^{(c)} := \sinh \left(1 + \frac{i\pi}{2}\right). \]

6. (Problems 2.14.7, 9 and 19, p. 74)
Find all complex values of each of the following functions.

\[ z^{(a)} := \ln \frac{1 + i}{1 - i}, \quad z^{(b)} := (-1)^i \quad \text{and} \quad z^{(c)} := \cos(\pi + i \ln 2). \]
7. (Problems 2.15.7, 10 and 13, p. 74)
   Find all complex values of each of the following inverse functions.
   \[ z^{(a)} := \tan^{-1}(i\sqrt{2}), \quad z^{(b)} := \cos^{-1}\frac{5}{4} \quad \text{and} \quad z^{(c)} := \cosh^{-1}(-1). \]

8. (Problems 14.1.8, 12 and 16, p. 667)
   Find the real and imaginary parts \( u(x, y) \) and \( v(x, y) \), respectively, of the following functions of a complex variable \( z = x + iy \).
   \[ f^{(a)}(z) := \sin z, \quad f^{(b)}(z) := \frac{z}{z^2 + 1} \quad \text{and} \quad f^{(c)}(z) := z^2 - \bar{z}^2. \]

9. (Problems 14.2.8, 12 and 16, p. 672)
   Use the Cauchy–Riemann conditions to determine all points \( z \) where each of the functions in the previous problem is analytic.

10. (Problems 14.2.36, 40 and 41, p. 667)
    Expand each of the following functions of a complex variable \( z = x + iy \) in a power series about the origin \( z = 0 \) in the complex plane. Find the disk of convergence for each.
    \[ f^{(a)}(z) := \sqrt{1 + z^2}, \quad f^{(b)}(z) := \frac{1}{1 - z} \quad \text{and} \quad f^{(c)}(z) := e^{iz}. \]