Problem Set I
Due: Tuesday, 31 August 2010

1. (Problems 1.10.5, 9 and 15, p. 22)
Find the interval of convergence of each of the following power series. Be sure to investigate the endpoints of the interval in each case.

\[ S(a) = \sum_{n=1}^{\infty} \frac{x^n}{(n!)^2}, \quad S(b) = \sum_{n=1}^{\infty} (-1)^n n^3 x^n \quad \text{and} \quad S(c) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n}. \]

2. (Problems 1.13.9, 13 and 17, p. 32)
Expand each of the following functions in a Maclaurin series (a Taylor series about \( x = 0 \)). Write the series as an infinite sum and calculate the first few (non-zero) terms explicitly.

\[ f(a)(x) = 1 + \frac{x}{1-x}, \quad f(b)(x) = \int_{0}^{x} e^{-t^2} \, dt \quad \text{and} \quad f(c)(x) = \ln \frac{1 + x}{1-x}. \]

3. (Problems 1.13.26, 31 and 35, p. 32)
Find the first few terms of the Maclaurin series for each of the following functions and check your results by computer.

\[ f(a)(x) = \frac{1}{\sqrt{\cos x}}, \quad f(b)(x) = \cos(e^x - 1) \quad \text{and} \quad f(c)(x) = \frac{x}{\sin x}. \]

4. (Problems 1.15.7, 12 and 23c, pp. 40–41)
Use Maclaurin series to evaluate the following limits.

\[ y(a) = \frac{d^8}{dx^8}(x^6 \tan^2 x) \bigg|_{x=0}, \quad y(b) = \lim_{x \to 0} \frac{\tan x - x}{x^3} \quad \text{and} \quad y(c) = \lim_{x \to 0} \left( \csc^2 x - \frac{1}{x^2} \right). \]

5. (Problem 1.16.28, p. 42)
The energy of an electron at speed \( v \) in special relativity theory is \( E = mc^2(1 - v^2/c^2)^{-1/2} \), where \( m \) is the electron mass, and \( c \) is the speed of light. The factor \( mc^2 \) is called the rest mass energy (the energy when \( v = 0 \)). Find the first three terms of the series expansion of \( E \) in \( v \). What is the second term in the series?

6. (Problem 1.16.33, p. 43)
If you are at the top of a tower of height \( h \) above the surface of the earth, show that the distance you can see along the surface of the earth is approximately \( s = \sqrt{2Rh} \), where \( R \) is the radius of the earth. (See the figure and the hints in the book.)
7. (Problem 4.12.16, p. 237)

In kinetic theory, we have to evaluate integrals of the form

\[ I(n) = \int_{0}^{\infty} t^n e^{-at^2} \, dt. \]

Given that \( I(0) = \sqrt{\pi/4a} \), evaluate \( I \) for all integers \( n \).