

Home Work 2
Due: September 21, 2004
PHY 6938 & PHY 4936
General Relativity

- I. Consider the line element $ds^2 = -dt^2 + dx^2$. Define two new variables, $u = x - t$ and $v = x + t$.
- Write the line element in the new variables, u and v .
 - Calculate the basis vectors e_u and e_v in terms of e_x and e_t .
 - Calculate the basis 1-forms w^u and w^v in terms of w^x and w^t .

- II. Let $\sigma = \sigma_\alpha \omega^\alpha = \frac{3}{4}\omega^u + \omega^v$ be a 1-form in the metric spacetime given in the previous problem in terms of the coordinate variables u and v (Problem I). The components of σ are $\{\sigma_\alpha\} = \{\sigma_u, \sigma_v\} = \{3/4, 1\}$.

- What are the covariant components of the metric, $g_{\alpha\beta} \omega^\alpha \otimes \omega^\beta$?
- Write down its inverse, i.e. the contravariant components of the metric, $g^{\alpha\beta} e_\alpha \otimes e_\beta$?
- What are the contravariant components of σ ?

$$\{\sigma^\alpha\} = \{\sigma^u, \sigma^v\} = ?$$

- III. This problem deals with the stress-energy tensor T for a swarm of non-interacting identical particles of rest mass m .

- Explicitly write the components of the stress-energy tensor for a swarm of non-interacting identical particles (dust) in the frame locally comoving (*lcf*) with the dust.
- Using the Lorentz transformation for an observer moving with velocity β with respect to this *lcf* frame (velocity along the x-coordinate), rewrite the components of the stress-energy tensor in this new frame.
- Show that the components of T that you calculated in part b) of this problem are consistent with the definitions given in class (or in MTW) for the stress-energy tensor ($T^{oo}, T^{oi}, T^{io}, T^{ij}, T^{ji}$).