

Final Exam
Due: December 7, 2004, by 18:30¹
PHY 6938
General Relativity

- 1 The basis 1-forms for a special set of observers around a rotating black hole spacetime (Kerr Spacetime) are given by:

$$\begin{aligned}\omega^{\hat{t}} &= |g_{tt} - \omega^2 g_{\phi\phi}|^{1/2} dt \\ \omega^{\hat{r}} &= (\Sigma/\Delta)^{1/2} dr \\ \omega^{\hat{\theta}} &= \Sigma^{1/2} d\theta \\ \omega^{\hat{\phi}} &= (g_{\phi\phi})^{1/2} (d\tilde{\phi} - \omega dt).\end{aligned}$$

In these expressions, $\omega \equiv -g_{t\phi}/g_{\phi\phi}$, and g_{tt} , $g_{\phi\phi}$, Σ and Δ are associated to the Kerr spacetime and can be found in MTW on page 877. These observers are referred to in the literature as “Zero Angular Momentum Observers,” or ZAMO’s for short. The 4-velocity ($u = e_{\hat{t}}$) of a ZAMO has no rotation.

- 1a) Show that in the Kerr spacetime, these basis 1-forms form an orthogonal set.
 - 1b) Calculate the dual basis vectors for the ZAMO’s.
 - 1c) Show that a ZAMO is not an inertial observer, i.e. calculate its acceleration.
- 2 Assume astronaut Eric is at rest in the cosmological comoving coordinates of a Friedmann universe ($\chi = \theta = \phi = \text{constant}$) with zero cosmological constant ($\Lambda = 0$). He shoots his laser pointer in some direction (please neglect the huge recoil Eric’s laser pointer will give him). In principle, can a photon from his laser pointer encircle the universe and return to hit the back of his laser pointer? What if there is a non-vanishing cosmological constant?
- 3 The most general second rank, symmetric tensor constructible from the Riemann and the metric, which is also linear in the Riemann, is

$$\begin{aligned} & a R_{\alpha\beta} + b R g_{\alpha\beta} + \Lambda g_{\alpha\beta} \\ = & a R^{\mu}{}_{\alpha\mu\beta} + b R^{\mu\nu}{}_{\mu\nu} g_{\alpha\beta} + \Lambda g_{\alpha\beta} \end{aligned}$$

where a , b and Λ are constants. Show that this tensor has an automatically vanishing divergence if and only if $b = -\frac{1}{2}a$.

¹A date which will live in infamy!