Physics 6938 | Florida Atlantic University
General Relativity | Fall, 2007

Problem Set I

Due: Thursday, 20 September 2007

1. [*d'Inverno* 2.6] In the event diagram of Fig. 2.14, find a geometrical construction for the world-line of an inertial observer passing through O who considers event G as occurring simultaneously with O. Hence describe the world-lines of inertial observers passing through O who consider G as occurring before or after O.

2. [*d'Inverno* 3.3] Prove that the Lorentz transformation in two (spacetime) dimensions can be written in the form

$$ct' = -x \sinh\phi + ct\cosh\phi \quad \text{and} \quad x' = x\cosh\phi - ct\sinh\phi,$$

where the *rapidity* ϕ is defined by $\phi = \tanh^{-1}(v/c)$.

Establish also the following version of these equations:

$$ct' + x' = \mathrm{e}^{-\phi}(ct + x), \quad ct' - x' = e^{\phi}(ct - x) \quad \text{and} \quad e^{2\phi} = \frac{c+v}{c-v}.$$

What relation does ϕ have to θ in equation (3.11)?

3. [*d'Inverno* 3.9] A particle moves from rest at the origin of a frame S along the x-axis, with constant acceleration α (as measured in an instantaneous rest frame). Show that the equation of motion is

$$\alpha x^2 + 2c^2 x - \alpha c^2 t^2 = 0,$$

and prove that light signals emitted after time $t = c/\alpha$ at the origin will never reach the receding particle.

A standard clock carried along with the particle is set to read zero at the beginning of the motion and reads τ at time t in S. Using the clock hypothesis, prove the following relationships:

$$\frac{u}{c} = \tanh\frac{\alpha\tau}{c}, \qquad \left(1 - \frac{u^2}{c^2}\right)^{-1/2} = \cosh\frac{\alpha\tau}{c}$$

$$\frac{\alpha t}{c} = \sinh\frac{\alpha\tau}{c}, \qquad x = \frac{c^2}{\alpha}\left(\cosh\frac{\alpha\tau}{c} - 1\right).$$

Show that, if $T^2 \ll c^2/\alpha^2$, then, during an elapsed time T in the inertial system, the particle clock will record approximately the time $T\,(1 - \alpha^2 T^2/6c^2)$.

If $\alpha = 3g$, find the difference in recorded times by the spaceship clock and those of the inertial system (a) after 1 hour and (b) after 10 days.

4. [*d'Inverno* 4.3] A particle of rest mass $\bar{m}_0$ and speed u strikes a stationary particle of mrest mass m_0. If the collision is perfectly inelastic, then find the rest mass of the composite particle.

5. [*d'Inverno* 4.8] A particle of rest mass m_0, energy e_0, and momentum p_0 suffers a head-on elastic collision (i.e., masses of particles unaltered) with a stationary mass M_0. In the collision, M_0 is knocked straight forward, with energy E and momentum P, leaving the first particle with energy e and momentum p. Prove that

$$P = \frac{2M_0 p_0 \left(e_0 + M_0 c^2\right)}{2M_0 e_0 + M_0^2 c^2 + m_0^2 c^2} \quad \text{and} \quad p = \frac{p_0 \left(m^2 c^2 - M^2 c^2\right)}{2M e_0 + M^2 c^2 + m^2 c^2}.$$

What do these formulae become in the classical limit?

6. [*d'Inverno* 4.11] An atom of rest mass m_0 is at rest in a laboratory and absorbs a photon of frequency ν. Find the velocity and mass of the recoiling particle.