The Principle of Relativity

Lecture I
General Relativity (PHY 6938), Fall 2007

FLORIDA ATLANTIC UNIVERSITY

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Why Spacetime?

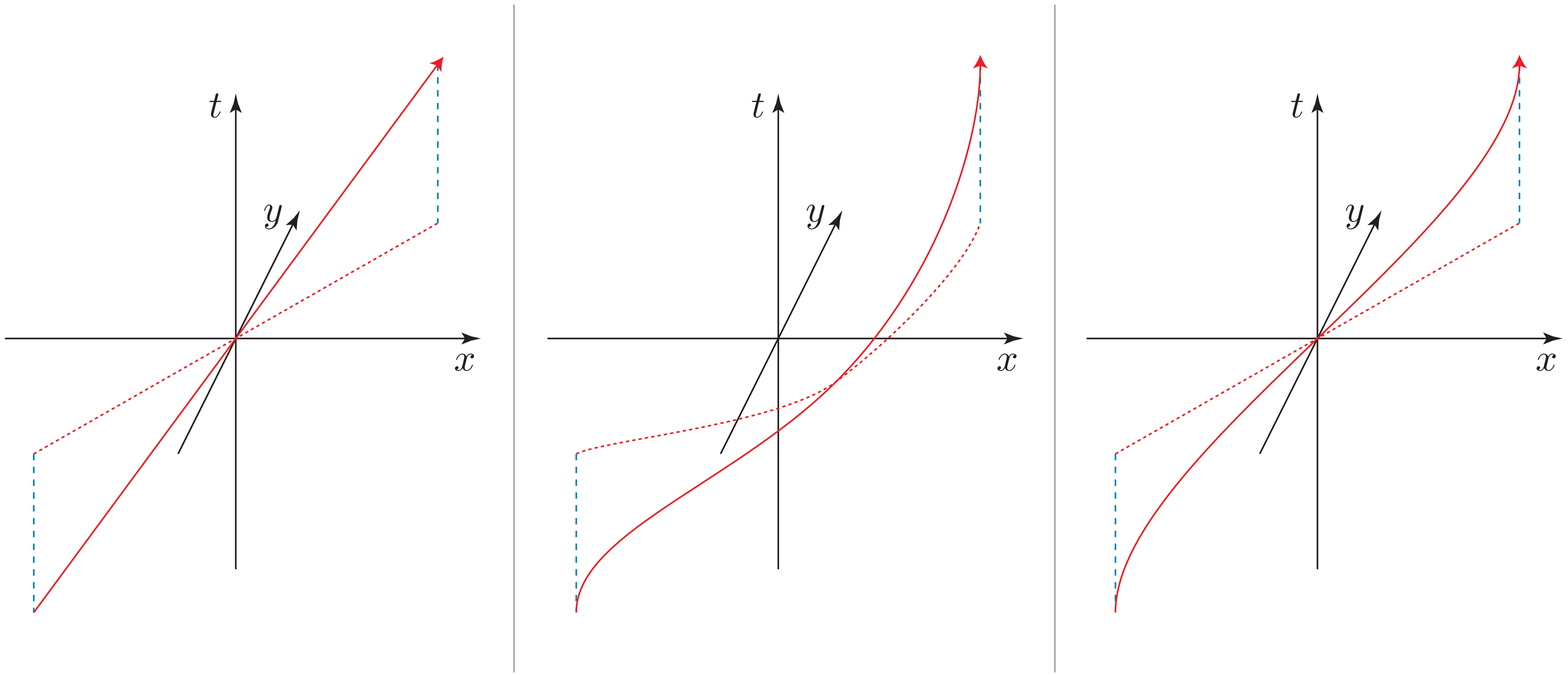
- Space and time are tied together in the idea of motion.

> Space and time are commonly regarded as the forms of existence in the real world, matter as its substance. A definite portion of matter occupies a define part of space at a definite moment of time. It is in the composite idea of motion that these three fundamental conceptions enter into intimate relationship.
>
> —Hermann Weyl (1921)

- There are no preferred points of space or moments of time, but there are preferred (inertial) motions through spacetime.

> Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.
>
> —Isaac Newton (1687)

- Newton is right to emphasize that there exist preferred, force-free motions, but he also makes an assumption about what they are: straight lines through Euclidean space.



Inertial motions are properties of spacetime.

Test Particles and Inertial Motions

- How can we observe inertial motions experimentally?
- A test particle is:
  - isolated (no contact forces),
  - small, non-spinning (no tidal forces),
  - electrically neutral (no electromagnetic forces), and
  - not massive (no gravitational radiation).
- Test particles do not couple to known, long-range forces, except for gravity, and so follow “natural” paths through spacetime.
- But, we must measure what these inertial motions are, not postulate them.

The Principle of Relativity

Through any point of space, at any moment of time, there is exactly one inertial motion for each initial velocity a test particle might have at that point. The fundamental laws of physics do not distinguish these motions.

Relativity and Electrodynamics

- Lorentz/Fitzgerald approach (Bell 1976)
  - Electric and magnetic fields of moving charges modify the dynamics of (classical) atoms. They get flatter and the orbital period gets shorter.
  - Therefore, using real rulers and (atomic) clocks, a moving observer will assign different positions and times to events than a stationary one.
- Einstein approach
  - The key result of these modified rulers and clocks is that all inertial observers see a light ray propagate through vacuum at the same speed *c*.

$$t' = t$$
$$\vec{r}\,' = \vec{r} - \vec{v}t$$

$$t' = \frac{t - \vec{v}\cdot\vec{r}/c^2}{\sqrt{1 - v^2/c^2}}$$
$$\vec{r}\,' = (\vec{r} - \hat{v}\hat{v}\cdot\vec{r}) + \frac{\hat{v}\hat{v}\cdot\vec{r} - \vec{v}\,t}{\sqrt{1 - v^2/c^2}}$$

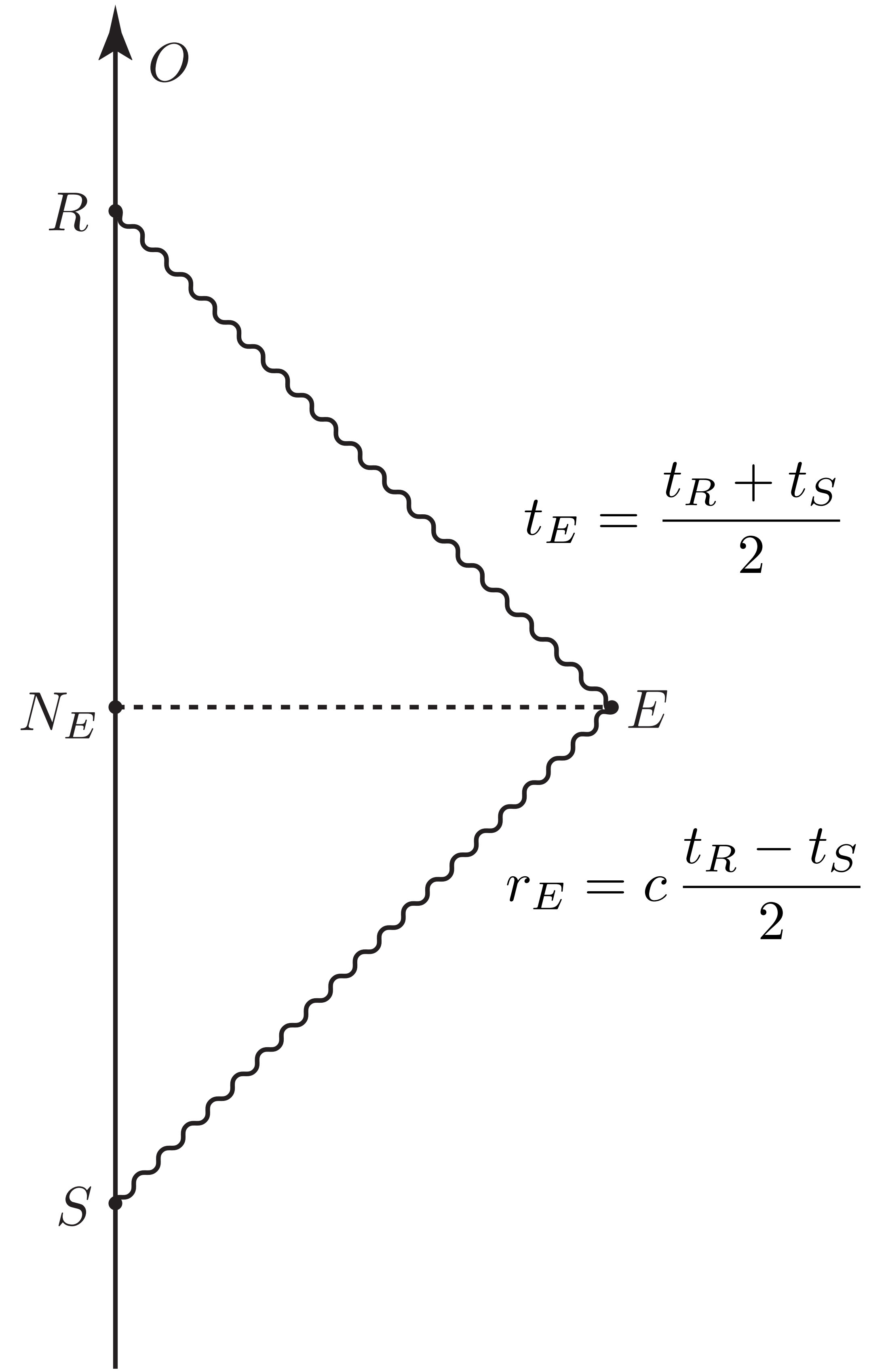
Principles and Assumptions of Special Relativity

- Principles: Relativity and Constant Speed of Light
- Assumptions:
  - Euclidean Space: Every inertial observer finds space at any moment of time to be a three-dimensional Euclidean continuum.
  - Rectilinear Motion: Every inertial observer finds that both other inertial observers and light rays move along straight lines at uniform speed.
  - Homogeneous Time: Every inertial observer measure every other inertial observer's clock to run at a uniform rate.
  - Homogeneous Space: The relationship between the spatial coordinates associated to a fixed event by two inertial observers is linear, with perhaps an additional, linear time dependence.

$$\left(\vec{r}\,' = \vec{r} - \vec{v}t \right)$$

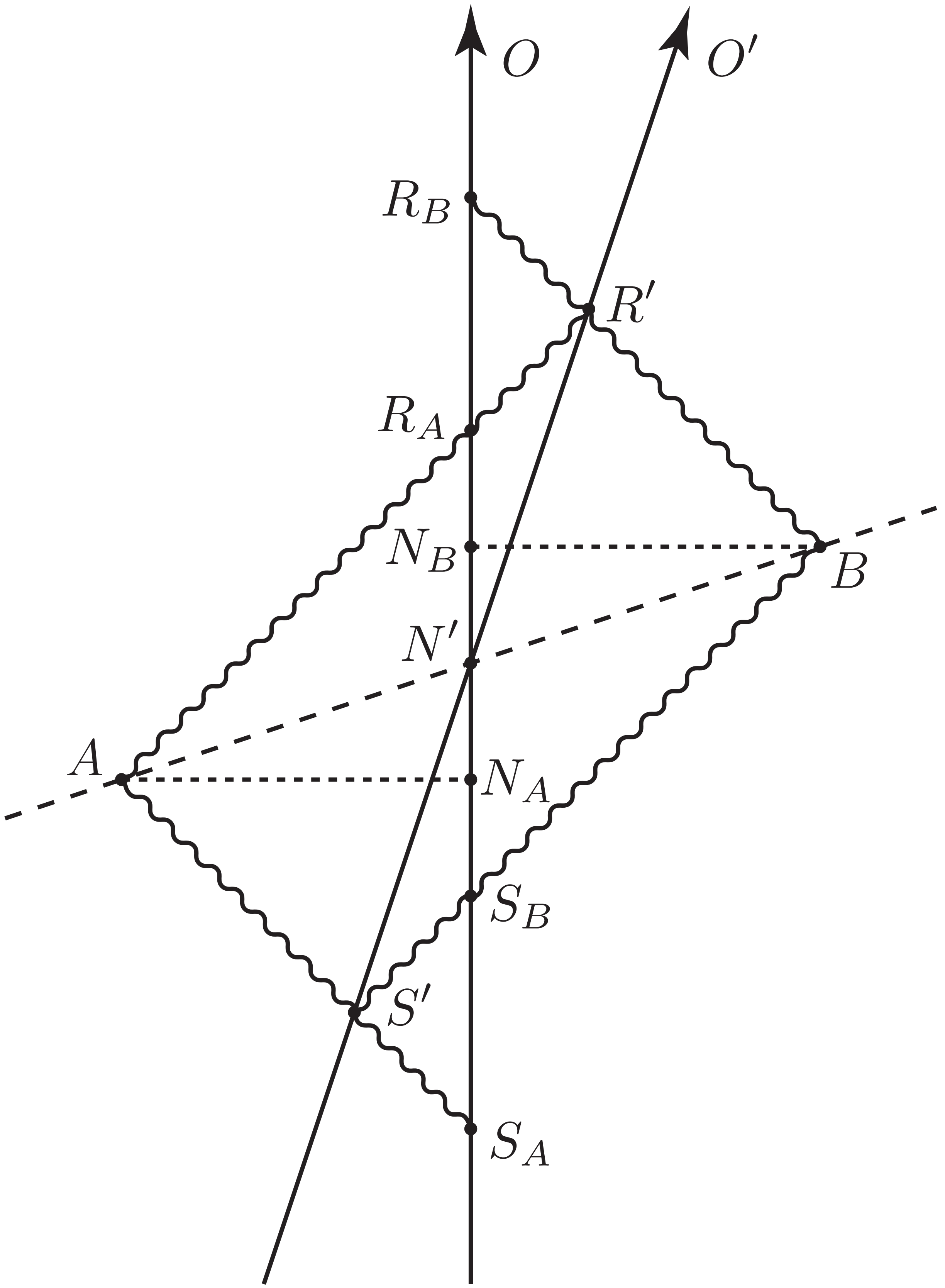
The Relativity of Simultaneity

- An inertial observer, equipped with a clock, can use light rays to locate remote events in spacetime.



The Relativity of Simultaneity

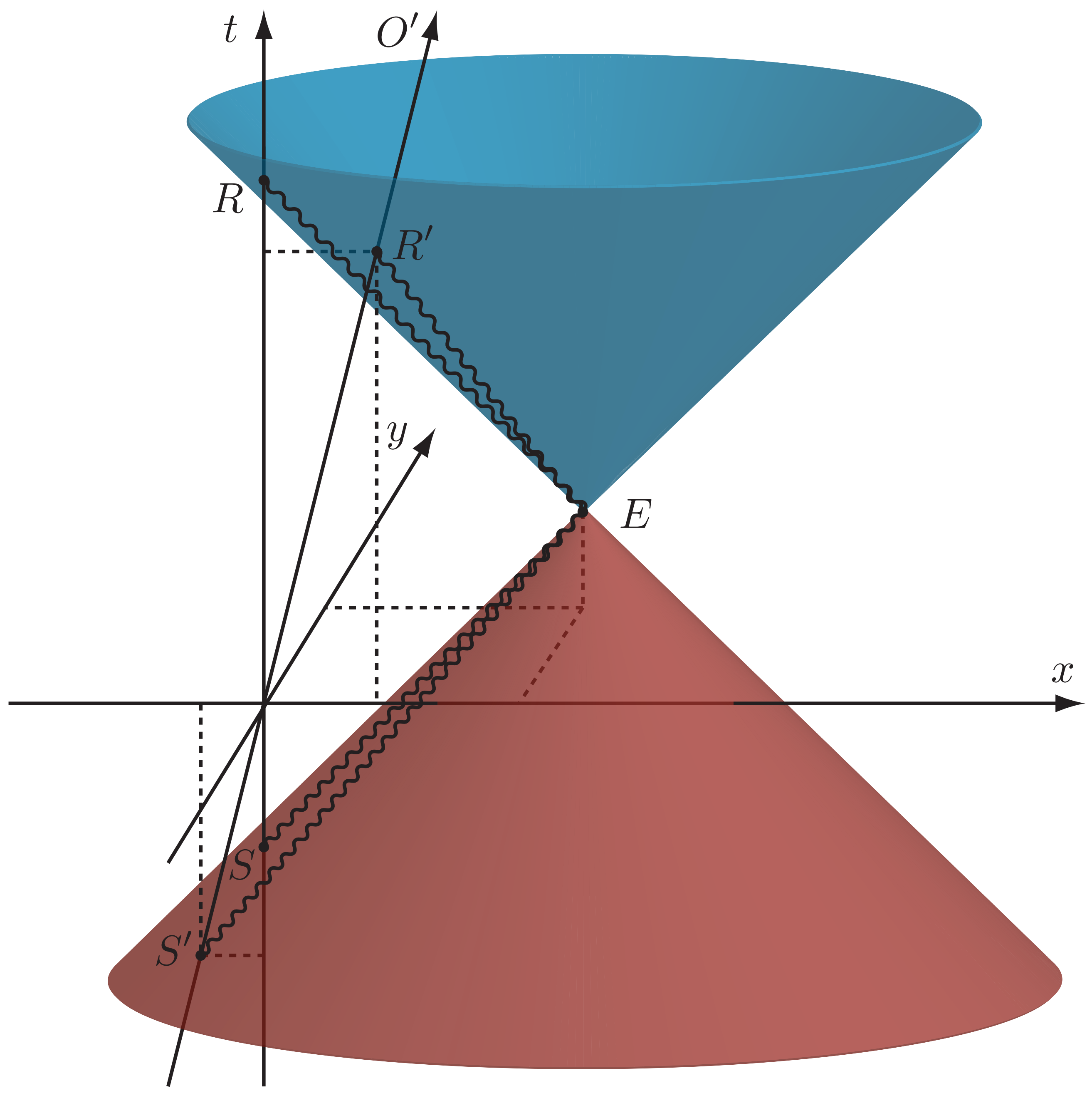
- An inertial observer, equipped with a clock, can use light rays to locate remote events in spacetime.
- Different inertial observers will not agree on whether two remote events occurred simultaneously or not.



Relationship Between Inertial Coordinate Systems

The relationship between the coordinates assigned to a fixed spacetime event by two different inertial observers is dictated by the relationship between the two pairs of times

$$t_S \text{ and } t_R \qquad \text{and} \qquad t'_{S'} \text{ and } t'_{R'}.$$



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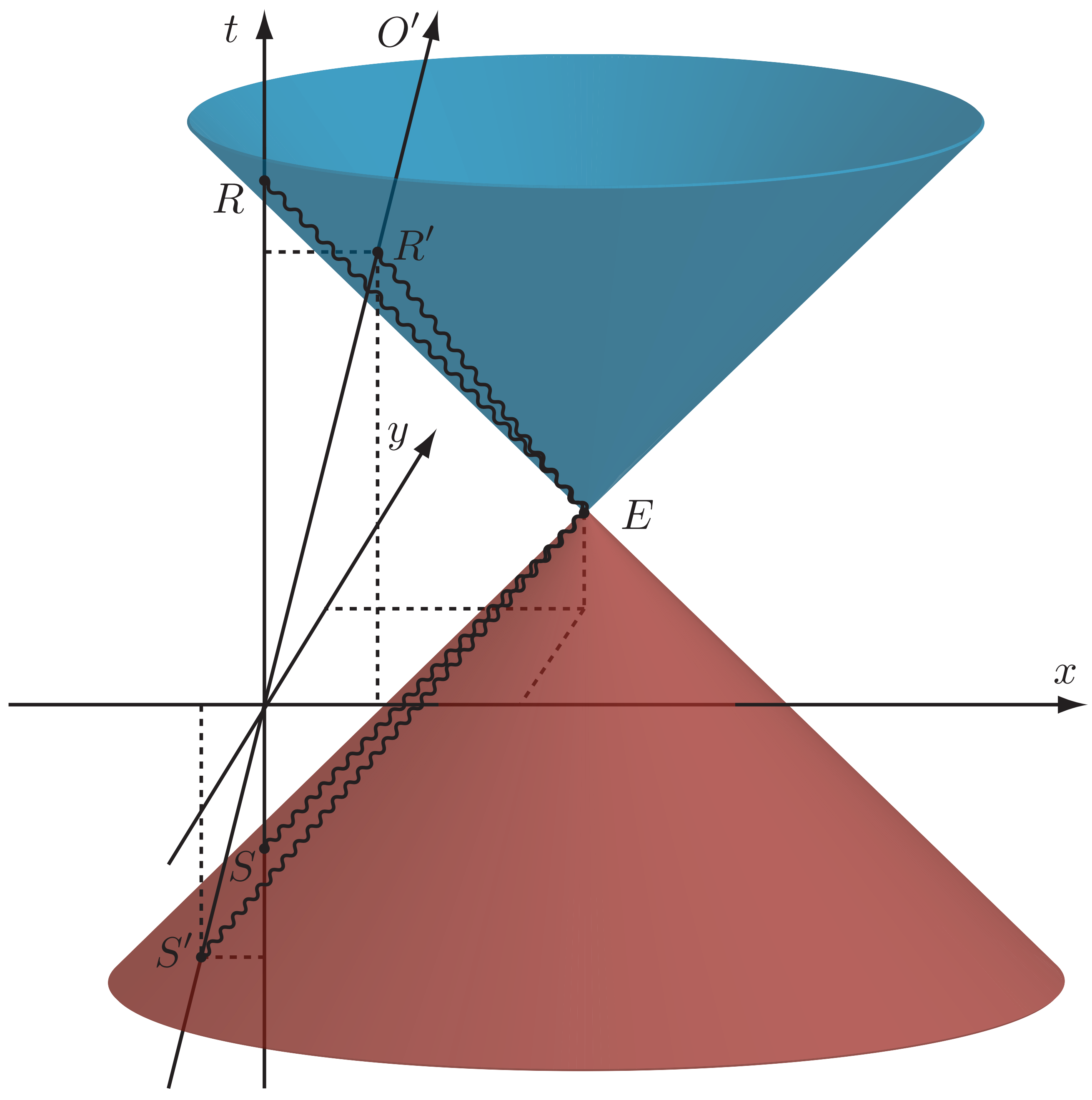

$$\left|\vec{r}_E - \vec{v}\,t\right|^2 = c^2\left(t_E - t\right)^2$$

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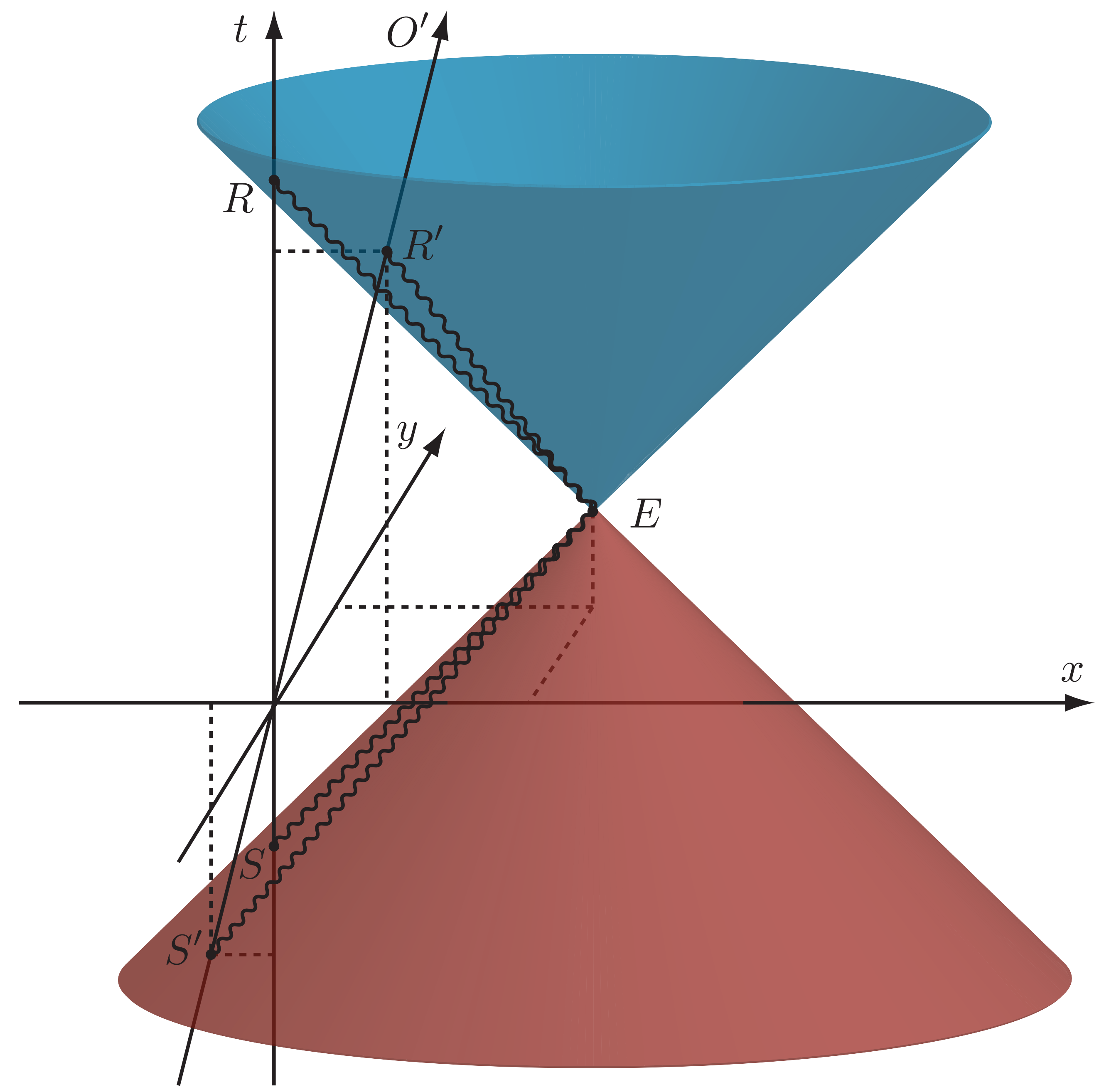

$$t^2 - 2\,\frac{c^2\,t_E - \vec{v}\cdot\vec{r}_E}{c^2 - v^2}\,t + \frac{c^2\,t_E^2 - |\vec{r}_E|^2}{c^2 - v^2} = 0 = (t - t_{S'})\,(t - t_{R'})$$

$$t_{S'} = \gamma\, t'_{S'} \quad \text{and} \quad t_{R'} = \gamma\, t'_{R'}$$

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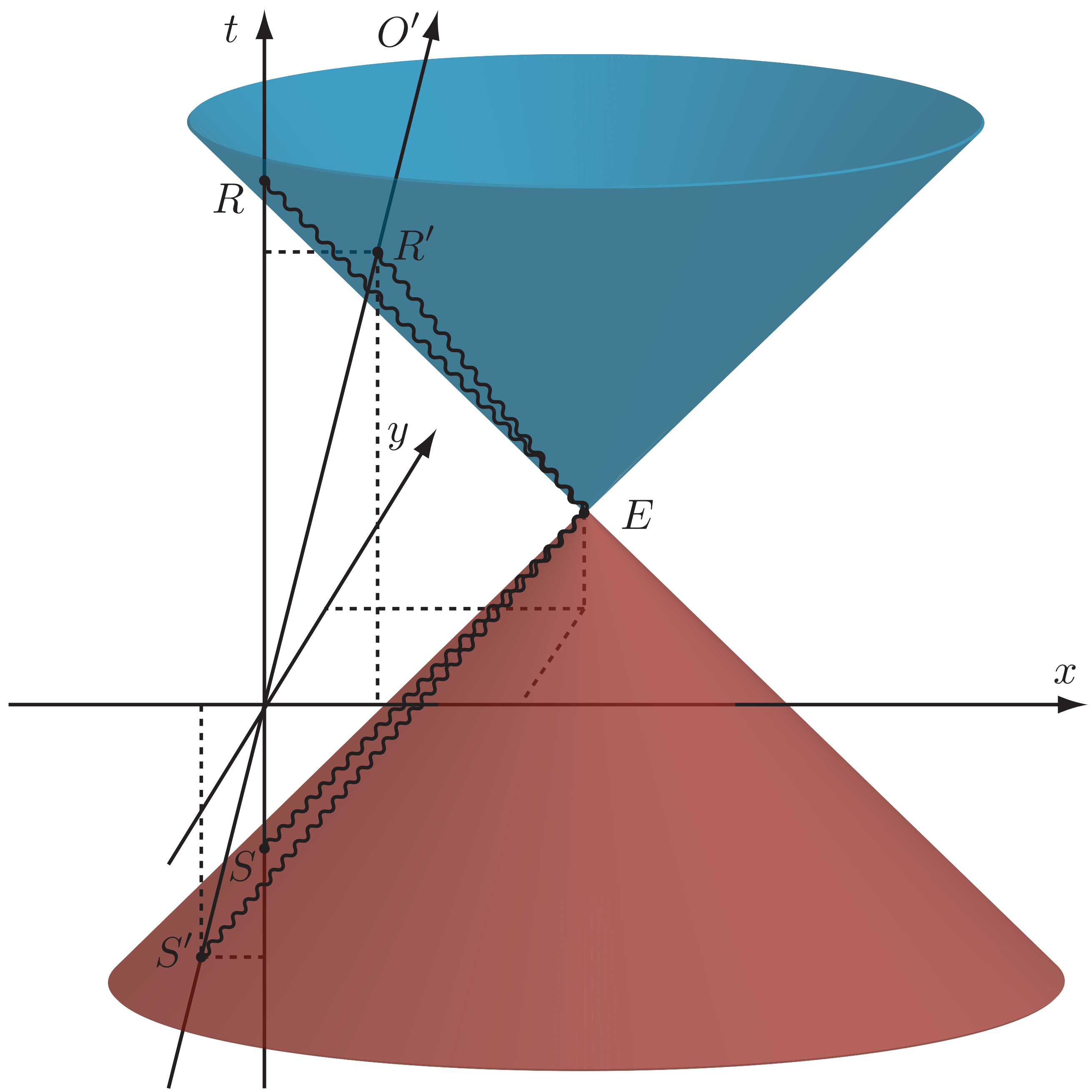

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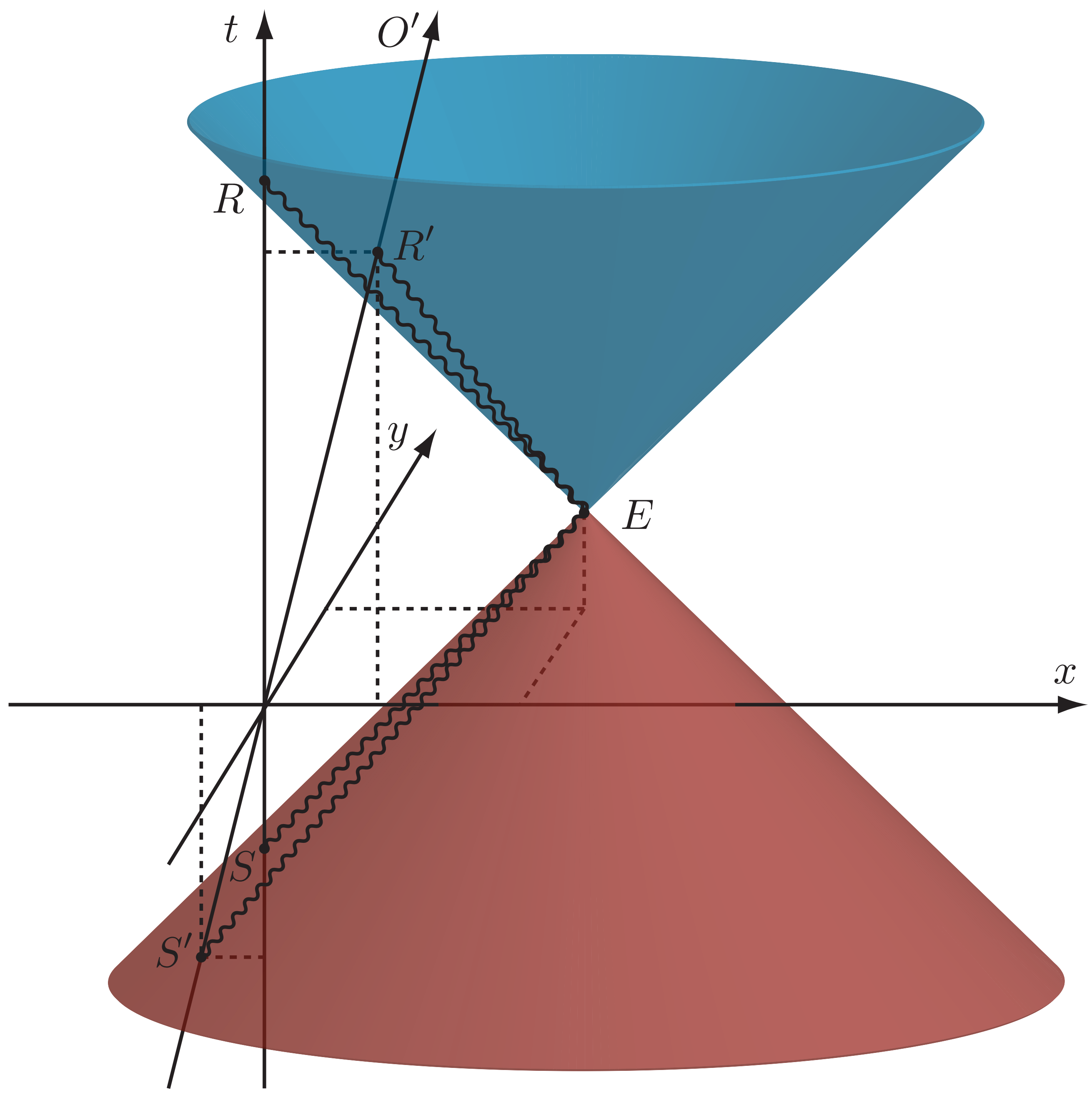

$$\gamma^2\, t'_{S'}\, t'_{R'} = t_{S'}\, t_{R'} = \frac{t_E^2 - \left|\vec{r}_E\right|^2 / c^2}{1 - v^2/c^2} = \frac{t_S\, t_R}{1 - v^2/c^2}$$

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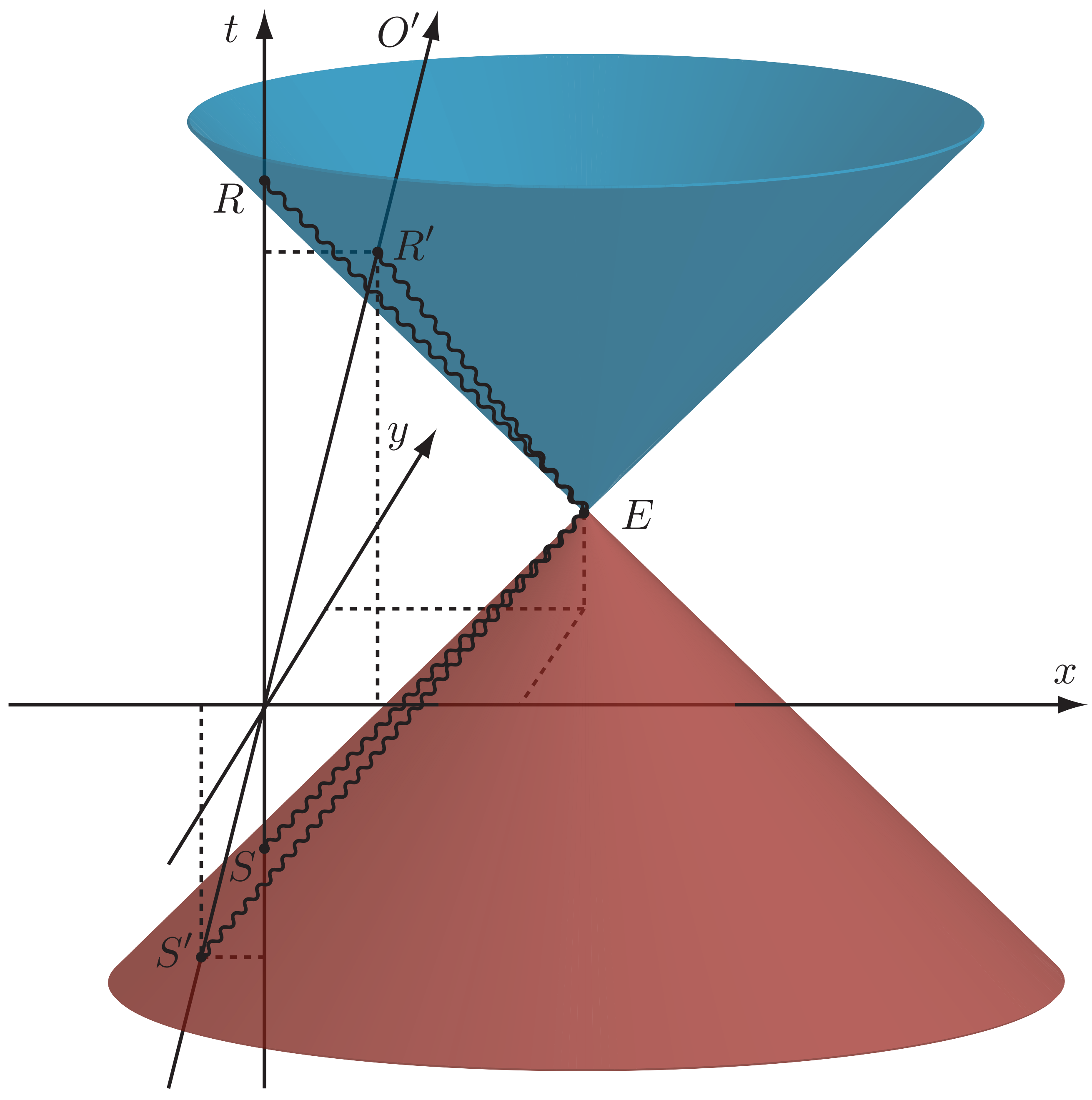

$$\gamma^2\, t'_{S'}\, t'_{R'} = \frac{t_S\, t_R}{1 - v^2/c^2} \qquad \text{and} \qquad (\gamma')^2\, t_S\, t_R = \frac{t'_{S'}\, t'_{R'}}{1 - v^2/c^2}$$

$$\gamma = \gamma'$$

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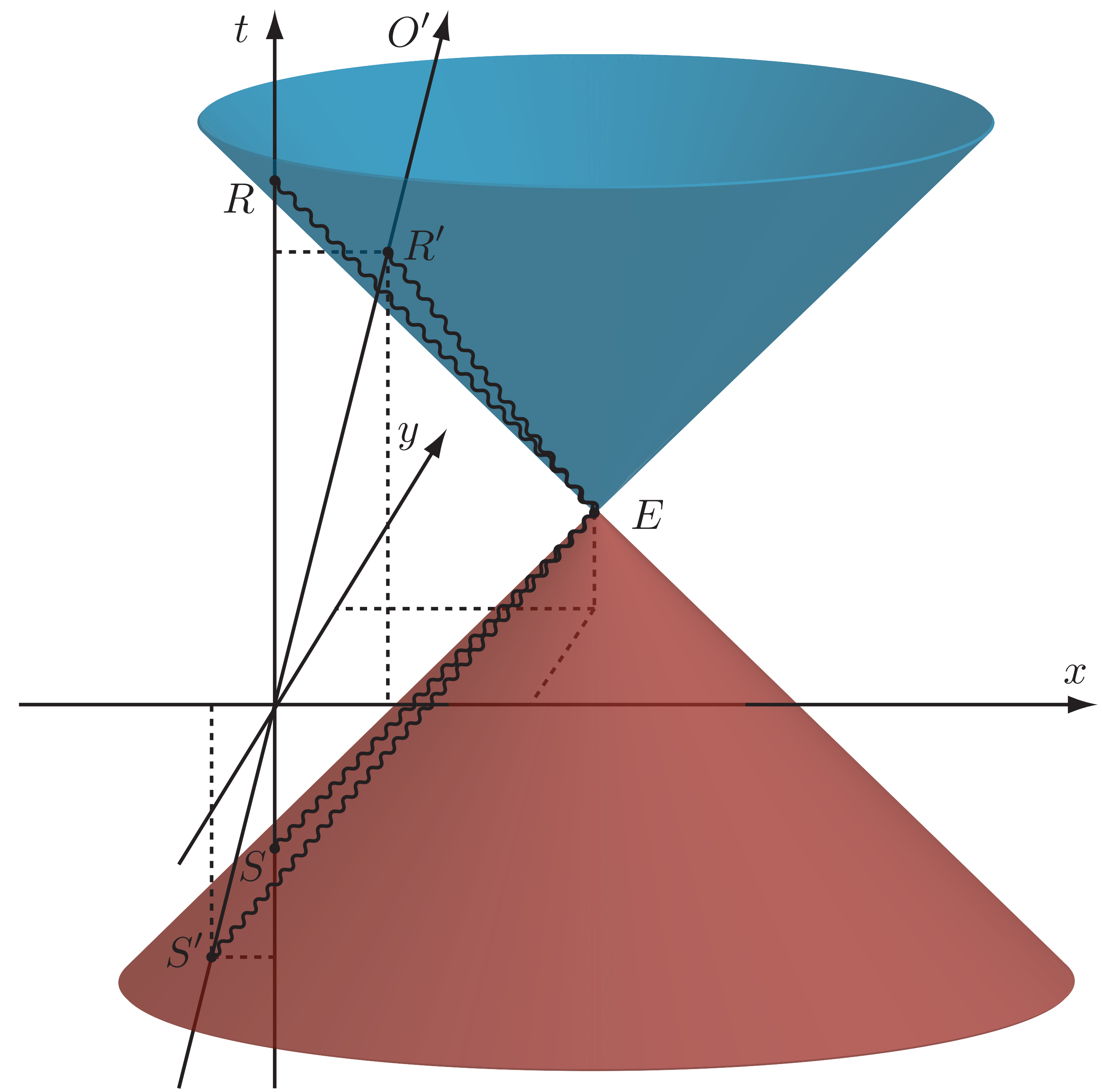

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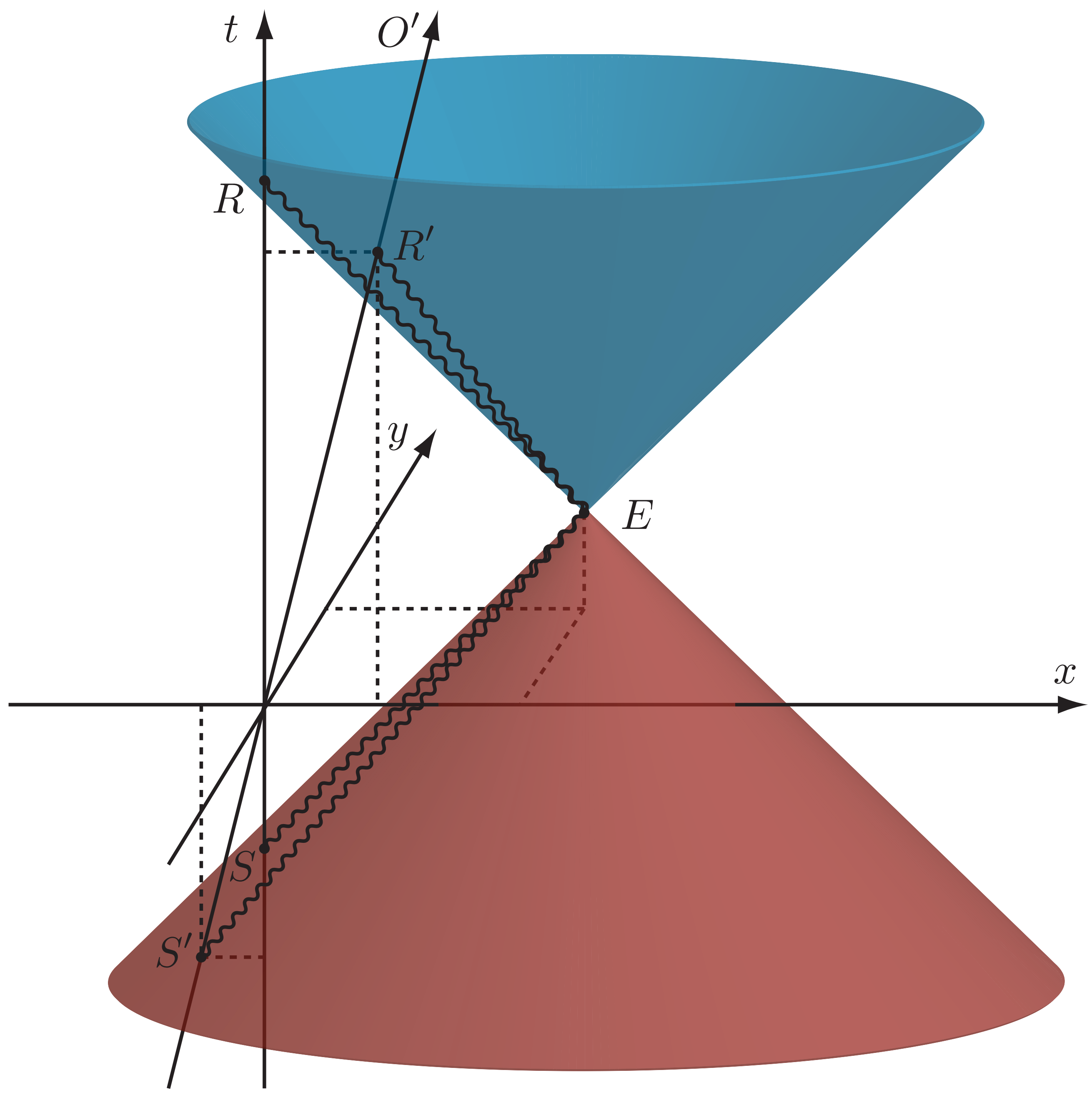

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \qquad \text{and} \qquad t'_{S'}\, t'_{R'} = t_S\, t_R$$

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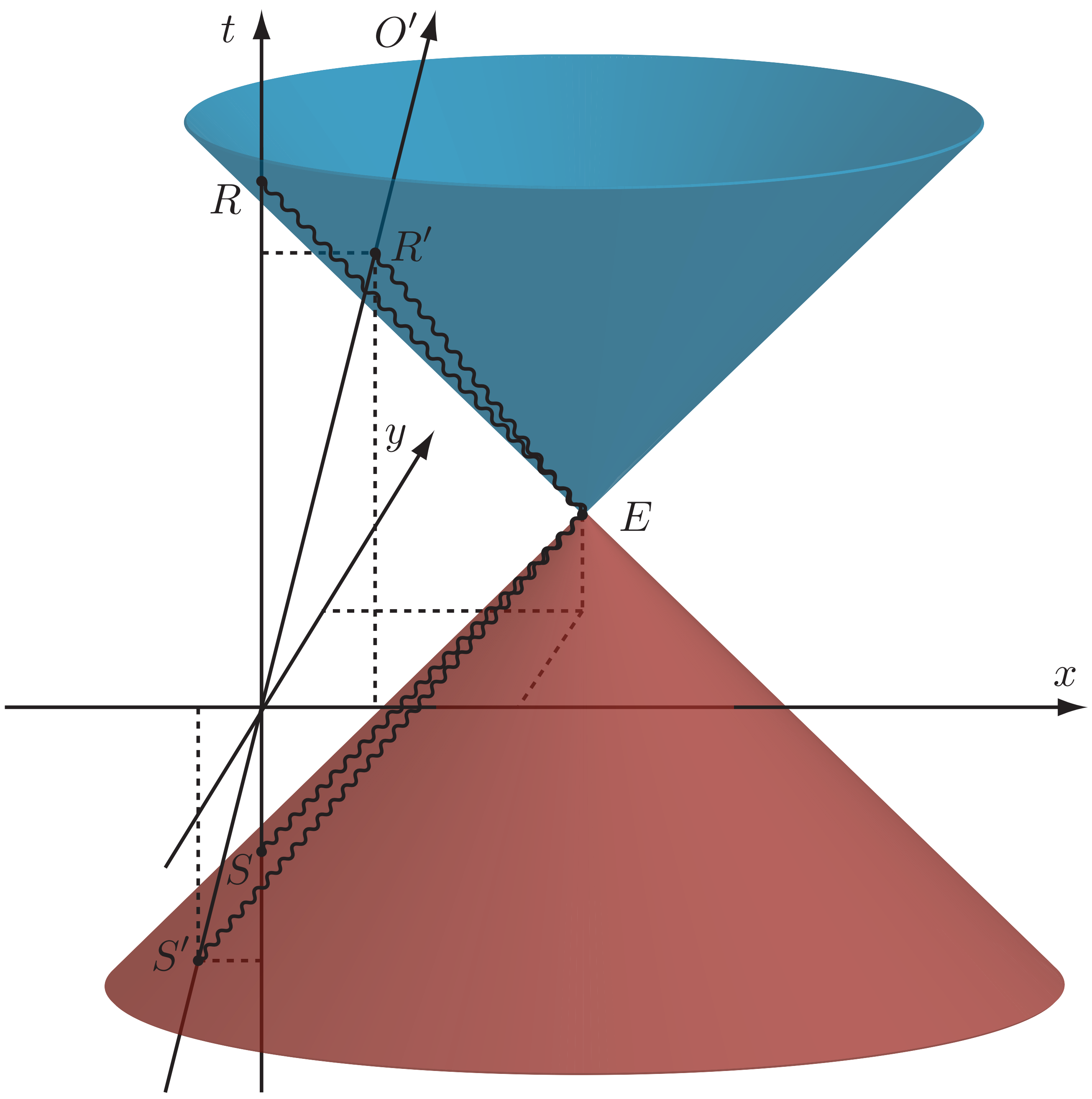

$$\|E\|^2 := -c^2\, t_S\, t_R = -c^2\,(t_E)^2 + |\vec{r}_E|^2$$

Lorentz Transformations

$$t^2 - 2\,\frac{c^2\,t_E - \vec{v}\cdot\vec{r}_E}{c^2 - v^2}\,t + \frac{c^2\,t_E^2 - |\vec{r}_E|^2}{c^2 - v^2} = 0$$

$$t'_E = \gamma\left(t_E - \vec{v}\cdot\vec{r}_E/c^2\right)$$

$$|\vec{r}\,'_E|^2 = |\vec{r}_E - \hat{v}\,\hat{v}\cdot\vec{r}_E|^2 + \gamma^2\left(\hat{v}\cdot\vec{r}_E - v\,t_E\right)^2$$

$$\begin{pmatrix} t' \\ \vec{r}\,' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma c^{-2}\,\vec{v}\,\cdot \\ -\gamma\,\vec{v} & I\,\cdot\, + (\gamma - 1)\,\hat{v}\hat{v}\,\cdot \end{pmatrix} \begin{pmatrix} t \\ \vec{r} \end{pmatrix}$$

- We can read off the time transformation law from previous results.
- The invariance of the interval then constrains the spatial variables.
- Linearity fixes the spatial part of the Lorentz transformation.

Linear Structure of Minkowski Spacetime

- Every inertial observer can define a vector structure on Minkowski spacetime using his or her own coordinates.

$$C = \alpha\, A + \beta\, B$$

where

- The Lorentz transformation of coordinates is linear, so all inertial observers define the same vector structure!

$$\begin{pmatrix} t_C \\ \vec{r}_C \end{pmatrix} := \alpha \begin{pmatrix} t_A \\ \vec{r}_A \end{pmatrix} + \beta \begin{pmatrix} t_B \\ \vec{r}_B \end{pmatrix}$$

- The Minkowski interval defines a quadratic form on this vector space. The associated bi-linear form is the Minkowski metric.

$$\boldsymbol{\eta}(\mathbf{x}, \mathbf{y}) := \mathbf{x} \cdot \mathbf{y} := -c^2\, t_x\, t_y + \vec{x} \cdot \vec{y}$$

Summary and Outlook

- Special relativity is not fundamentally different from previous physical theories. It merely asserts that the principle of relativity applies to optical experiments, as well as to mechanical.
- The main difference between Newtonian and special relativity is that time is relative: different inertial observers have different notions of simultaneity.
- Special relativity makes assumptions about the relative motion of inertial observers, which are in fact exactly the same assumptions Newton made. These assumptions are violated experimentally because nothing, not even a test particle, can be shielded from gravity. (equivalence principle)
- Nonetheless, the assumptions of special relativity imply that spacetime is a vector space equipped with a flat metric. These results are intimately connected to the rectilinearity of inertial motions in the absence of gravity. In a gravitational field, we should expect to lose both linearity and flatness.